Chandra Science Center

Grating Tool Specifications
Level 1.5:
LETG/HESF Region Mask Definitions
&
Wavelength Determination

David Huenemoerder

(http://space.mit.edu/ASC/docs/TBD)

Revision 2.0—4 Jun 1999

Document and Change Control Log

Date	Version	Section	Status
30 June 98	1.0	all	Initial version
1 June 99	2.0	Appendix B	Added developers' specifications.

Unresolved Issues

The following is a list of unresolved, un-reviewed, or un-implemented items:

1. need to move reference data to ARD ICD

Contents

1 Introduction

This memo specifies how region masks can be defined for filtering data taken with the LETG in conjunction with the High Energy Suppression Filter (HESF, also known as the Drake Flat). The regions are used to assign identifiers to photons according to which part of the spectrum they fell, as specified in the Grating Level 1.5 ICD[1]. Subsequent processing uses the region information in assigning wavelengths to photons. This transformation will also be outlined in this document.

2 Region Masks

The HESF is comprised of two facets which reflect the incident beam and divide it into two segments. Since there is a hole in the middle of the HESF, each of the two reflected spectra are discontinous. In addition, there is a non-reflected portion which falls in the same location as a non-HESF observation. Figure 1 shows the

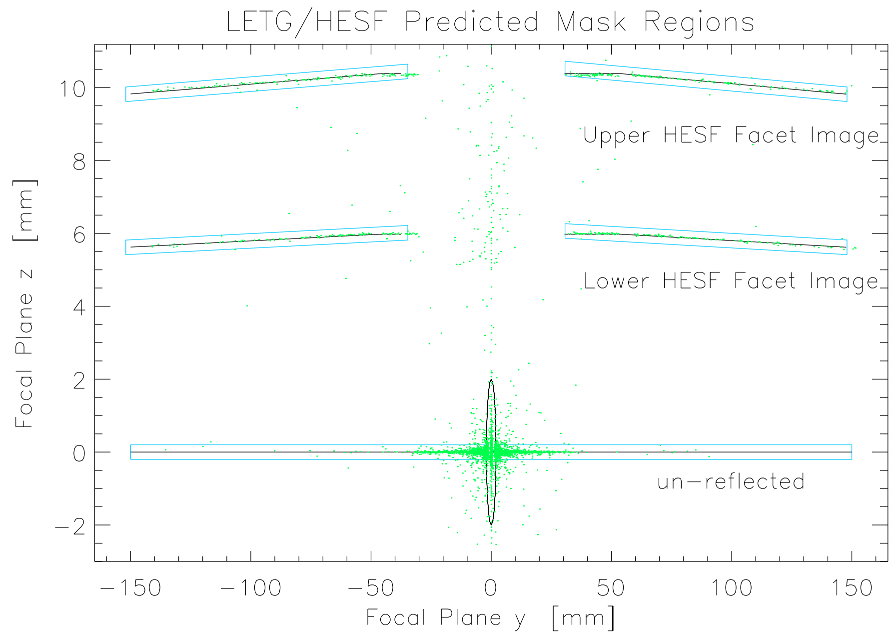


Figure 1: *The predicted regions defining the masks are shown by boxes, in which the predicted central locus of photons are shown by a solid line, and the photons positions from a* Marx *simulation are shown by discrete points. This view is in detector focal-plane coordinates (y, z) whose origin is the optical axis. No aspect dither motion was applied in this simulation.*

HRC-S field with regions outlined by boxes, the predicted central loci of photons, and results of a Marx simulation.

The locations of the spectrum for any aim-point and source position are determined by the HESF geometry and the HRC-S geometry. The tilts of the reflected spectra are caused by the HRC-S outer micro-channel plates' (MCP) tilts. Since

[1] "Grating Data Products: Level 1.5 to ASC Archive Interface Control Document", `http://space.mit.edu/ASC/docs/ICD_L1.5.ps.gz`

the optical axis is not on the detector centerline, and since the tilts of the $+y$ and $-y$ sides are not identical, the regions are not exactly symmetric.

The memo by Drake[2] derives the spectral locations from the HESF and HRC-S geometry. I have used his equations 4 and 5[3] to derive a single linear expression of the form

$$z = A_i + B_i y \tag{1}$$

which defines the focal-plane z position of the reflected spectrum given the focal-plane y-coordinate. It is actually a piecewise linear equation, since some of the coefficient terms have several discrete values, depending on the facet and on y. Keeping Drake's notation, the coefficients become:

$$A_i = \tan 2\theta_i \left[h_u \left(1 - \frac{\tan\theta_l}{\tan\theta_i} \right) + \left(\frac{z_p - z_f}{\tan\theta_i} \right) + h_l(0) + y_0(y) \tan\delta(y) \right] + z_s, \tag{2}$$

and

$$B_i = -\tan 2\theta_i \, \tan\delta(y). \tag{3}$$

Here, θ_i is to be replaced with θ_l or θ_u for the lower or upper facets, respectively. New terms I have introduced are:

$h_l(0)$**,** which is Drake's $h_l(y)$, evaluated at $y = 0$;

$y_0(y)$**,** which is the y-coordinate at which the MCP boundaries occur, and at which a tilt begins. This term has two values, one for each MCP boundary;

$\delta(y)$**,** which defines the tilt of the outer MCPs. This takes on one of three values, depending upon which MCP the value of y implies.

z_s**,** which is the zero-order source z-centroid. This changes the coordinate system from a zero-order referenced system to an optical-axis referenced system. Hence, the optical axis is by definition at $(y, z) = (0, 0)$.

A and B completely specify the locations of the spectrum and are sufficient to define a region mask around the spectrum. A, however, depends on observation-specific parameters: the SIM z-position, and the source z location, which both specify z_p. Hence, we could separate A into the "calibration" portion (A_{ci}) and the "observational" portion (A_{oi}) such that

$$A_i = A_{ci} + A_{oi} \tag{4}$$

as follows:

$$A_{ci} = \tan 2\theta_i \left[h_u \left(1 - \frac{\tan\theta_l}{\tan\theta_i} \right) - \left(\frac{z_f}{\tan\theta_i} \right) + h_l(0) + y_0(y) \tan\delta(y) \right], \tag{5}$$

$$A_{oi} = z_p \frac{\tan 2\theta_i}{\tan\theta_i} + z_s \tag{6}$$

To a good approximation, the ratio if tangents is equal to $2(1 + \theta_i^2)$. So in terms of the SIM z-offset, z_0, and zero-order centroid, z_s, we can write

$$A_{oi} \approx 2(1 + \theta_i^2) z_0 - (1 + 2\theta_i^2) z_s. \tag{7}$$

Note that this is the **only** *parameter of the reflected spectrum location which depends upon observational quantities!* The other terms are "just" calibration.

[2] "Locations of the Reflected Spectra from the HRC-S High Energy Suppresion Filter", Jeremy Drake, AXAF Science Center Memorandum, November 2, 1997

[3] There is an error in Equation 5: a factor of $\tan 2\theta_u$ should multiply the quantity in [ ].

All the terms, their definitions, and default values are given in the following table (see the Drake memo for defining diagrams).

Term	Nominal Value	Description
z_p	6.25 mm	The distance in the z-axis between the center of the HRC-S and the observation mean aim-point (and location of the zeroth order).
z_f	5.35 mm	The distance in the z-axis between the center of the HRC-S and the lower edge of the filter.[4]
θ_l	4.5°	the angle in the $x-z$ plane between the optical axis (x) and the plane of the lower facet.
θ_u	7.0°	The angle in the $x-z$ plane between the optical axis (x) and the plane of the upper facet.
h_u	22.3 mm	The x axis distance between the lower edge of the upper facet (nearest the HRMA) and the lower edge of the lower facet.
$h_l(0)$	26.3 mm	The x axis distance (or "height") of the lower edge of the lower facet (furthest from the HRMA) above the MCP surface, at $y=0$.
$y_0(y)$	-46 mm, $y \leq -46$ 54 mm, if $y > 54$ 0, otherwise (irrelevant)	The "boundaries" of MCP segments, relative to optical axis (which is different from the array centerline). These mark the locations at which the MCP tilts begin and $h_l(y) \neq h_l(0)$.
$\delta(y)$	$-1.23°$, $y \leq -46$ mm $0.0°$, $-46 < y \leq 54$ mm $1.38°$, $y > 54$	The tilt angle of the MCP (which which puts the y into $h_l(y)$). The negative is a convention to get the proper signs on the negative-y side. Note the y-limits should be the same as the values of $y_0(y)$.
y_{gm}	-36.7 mm	The HESF central gap limit on the minus-y side.
y_{gp}	28.7 mm	The HESF central gap limit on the plus-y side.
z_0	6.25 mm	The z-distance from the optical axis (by definition at $z=0$) to the HRC-S centerline. This is determined by the SIM z-offset.
z_s	0.0 mm	The z-coordinate of the zero-order centroid. This is nominally at the on-axis position, but may deviate from it. Given dither, a mean value is used.

Table 1: Parameters which determine the HESF spectral region.

The three observationally variable z's are related via

$$z_p = z_0 - z_s. \tag{8}$$

For the tabulated nominal values, the coefficients become:

[4]If there is mis-alignment of the HESF with the y-axis, then z_f becomes a funtion of y: $z_f(y) = z_{f0} + z_{f1}y$. If z_{f1} is non-zero, A_i will keep the same form, but B_i will have an additional term.

y range [mm]	A_l	B_l	A_u	B_u	A_{cl}	A_{ol}	A_{cu}	A_{ou}
$y \leq -46.0$	6.133	0.00340	10.627	0.00535	-6.445	12.558	-2.064	12.671
$-46.0 < y \leq -36.7$	5.977	0.000	10.381	0.000	-6.601	12.558	-2.310	12.671
$-36.7 < y \leq +28.7$	0	0	0	0	0	0	0	0
$+28.7 < y \leq +54.0$	5.977	0.000	10.381	0.000	-6.601	12.558	-2.310	12.671
$y > +54.0$	6.183	-0.00382	10.705	-0.0060	-6.395	12.558	-1.986	12.671

Table 2: Coefficients for nominal pointing.

3 Making a Mask

The previous sections described the locations of the spectrum in terms of focal-plane coordinates. Filtering is done, however, more easily in sky coordinates since the aspect correction restores the zero-order image. The purpose of the mask is to enclose photons of interest. Hence, the boundaries to not need to undergo rigorous coordinate transformations. Hence, we can simply rotate, translate, and scale the detector-plane coordinates to approximate sky coordinates.

There are also some simplifying factors for specifying the HESF regions. First, the zero-order will be near the on-axis point, so we do not need to consider far off-axis cases. Second, you will notice that there is no zero-order y-dependence in the reflected spectral positions — it is only the z-offset which determines where the reflected spectra lie.

The procedure for defining the regions is as follows:

1. Determine the mid-points between the gap inner edge and the array edge, on both the plus and minus sides. These won't change – they depend only on the calibration. These are:

$$y_{0p} = \frac{1}{2}\left(\max(y) + y_{gp}\right);$$

$$y_{0m} = \frac{1}{2}\left(\min(y) + y_{gm}\right).$$

2. Calculate the values of z at these values of y for the upper and lower facets:

$$z_{up} = A_u + B_u y_{0p},$$

$$z_{um} = A_u + B_u y_{0m},$$

$$z_{lp} = A_l + B_l y_{0p},$$

$$z_{lm} = A_l + B_l y_{0m}.$$

3. Define the centers of rectangular regions with these coordinates:

$$(y_{0p}, z_{up}),$$

$$(y_{0p}, z_{lp}),$$

$$(y_{0m}, z_{um}),$$

$$(y_{0m}, z_{lm}).$$

4. Assign a width to the rectangular region. Widths are specified via error-budget calculation as in non-HESF Level 1.5 processing (incorporating PSF vs off-axis angle lookup and defocus, astigmatism, and an arbitrary multiplication factor); typical widths are about ± 0.2 mm.

5. Assign a length to the rectangular region. The length need not be precise. It should extend in the y-direction from the central HESF gap to the detector edge, or about

$$L_p \approx \max(y) + y_{gp};$$

$$L_m \approx -(\min(y) + y_{gm}).$$

6. Determine the rotation of each rectangle. These do not depend upon any observational parameters, but only on calibration parameters. The rotations are given by the B_i values:

$$\rho_u = \arctan(B_u);$$

$$\rho_l = \arctan(B_l).$$

   (Remember that each of these has two values, due to the asymmetric MCP tilt angles, $\delta(y)$).

7. Transform the centers, widths, lengths, and rotations to the sky frame.

The predicted positions for the nominal parameters are are shown in Figure 1 for the entire field, and in Figure 2 with expanded scales.

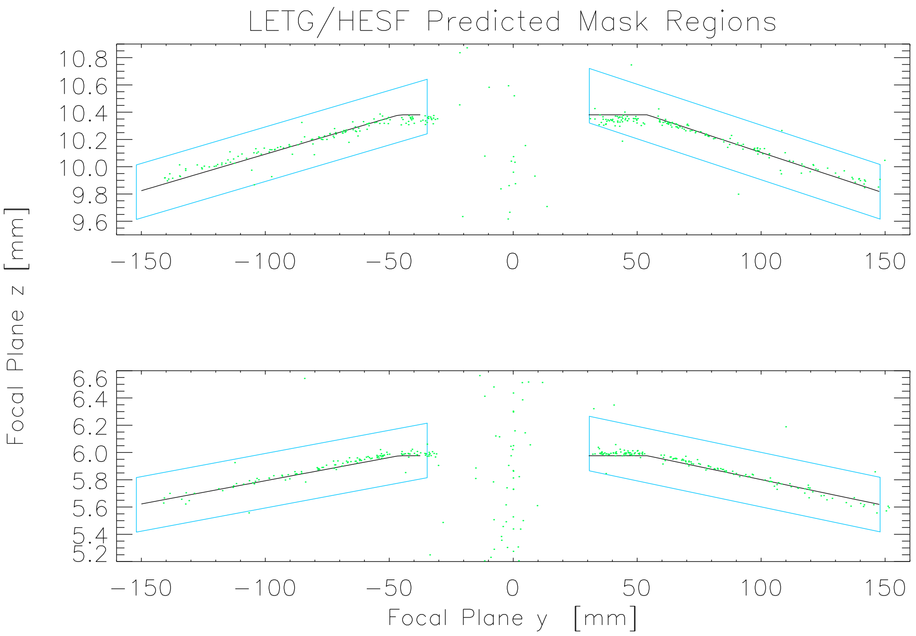


Figure 2: *The predicted regions defining the masks are shown by boxes, in which the predicted central locus of photons are shown by a solid line, and the photons positions from a* MARX *simulation are shown by discrete points. This view is in detector focal-plane coordinates (y, z) whose origin is the optical axis. No aspect dither motion was applied in this simulation.*

4 Diffraction Coordinate Specification

For non-HESF observations, diffraction coordinates are computed by transforming the 3D location of each event to the diffraction angles relative to the grating node parallel (r) and perpendicular (d) to the dispersion. The zero-order centroid is the origin for r, and the line through the zero-order centroid perpendicular to the grating bar direction is the reference for d.

With the HESF, the y-coordinate is unchanged, since the HESF diverts the photons perpendicularly to the dispersion direction.[5] Hence, the wavelength can be calculated as if the photon were in the un-reflected region.

The cross-diffraction angle must be referenced to the line, $z = A_i + B_i y$. The difference in cross-dispersion angle between any two photons will be the same whether they were reflected by the HESF, or not, but for an inversion in position. Hence, we can "fool" the current coordinate transformations by transforming from the deviation from the reflected line as if it were the deviation from the un-reflected line:

$$z' = (A_i + B_i y) - z + z_s \tag{9}$$

Here, the photon must first be tested for inclusion in region i (which is u or l for upper-facet or lower-facet reflection, respectively). Then it is transformed from (y, z') to (r, d) (the x-position is also the same as if it were un-reflected, to within mis-alignment affects).

[5]...if the HESF alignment is parallel to the y-axis. The magnitude of the misalignment affect is TBD.

A Examples *vs.* Source & SIM Positions

The following tables and figures are for different source off-axis positions or for different SIM z-offsets from the nominal parameters. There is currently a difference between the MARX coordinate system and that in the Drake memo. The former has the HESF at positive z if the optical axis is at the center of the HRC-S, and thus requires a translation toward $-z$ to enable the HESF. The Drake memo shows the opposite case. To reconcile the two, I have negated the MARX z-coordinates (the `zpos` vector) before plotting on the predicted mask positions. The relationship between the MARX parameters and HESF parameters defined here is as follows:

z_0: $-$DetOffsetZ

z_s: (SourceElevation/60) $(\pi/180)\, X_{\mathrm{HRMA}}$

y_s: ($-$SourceAzimuth/60) $(\pi/180)\, X_{\mathrm{HRMA}}$.

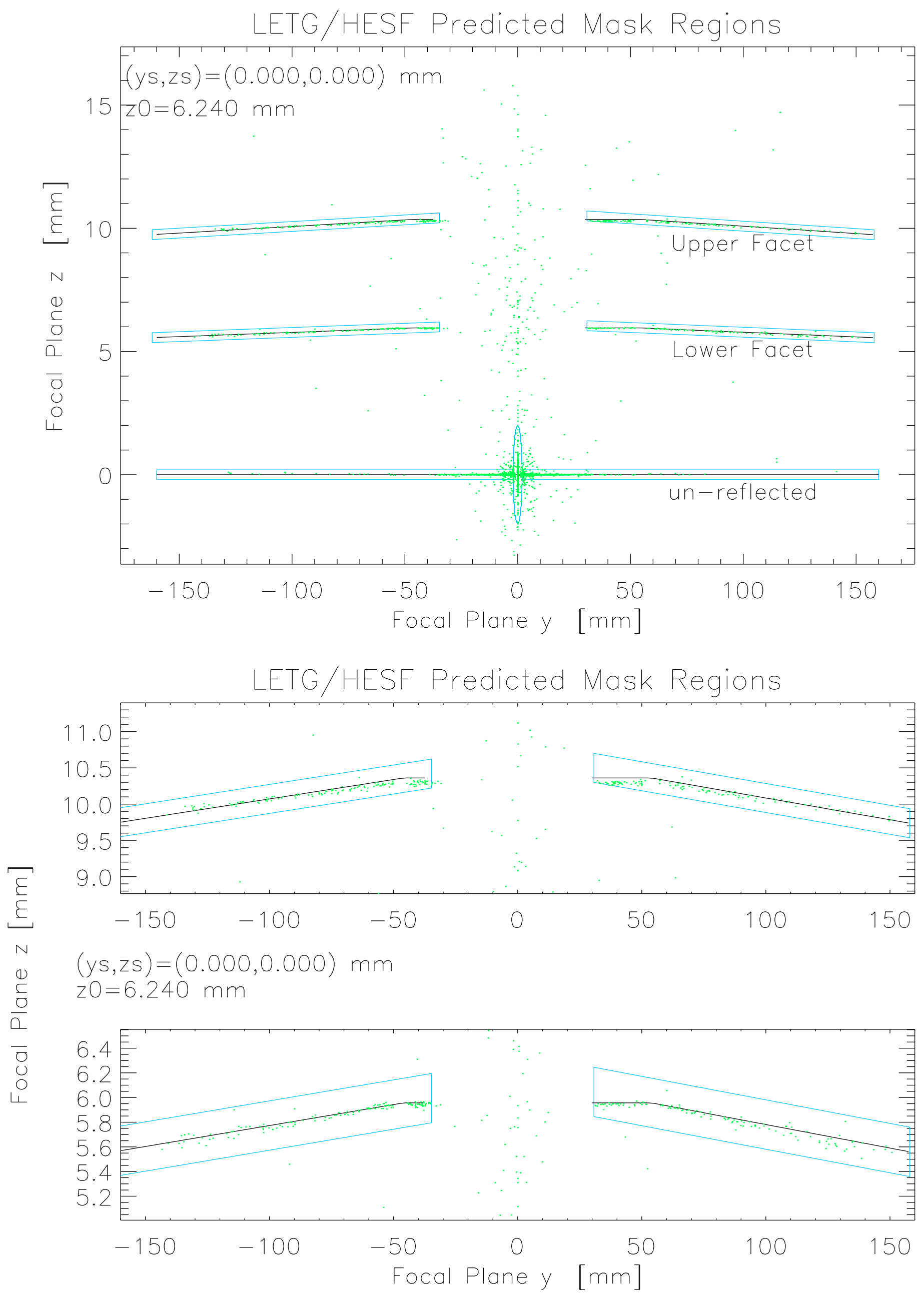
Here, X_{HRMA} is the imaging focal-length of the HRMA (about 10065.5 mm).

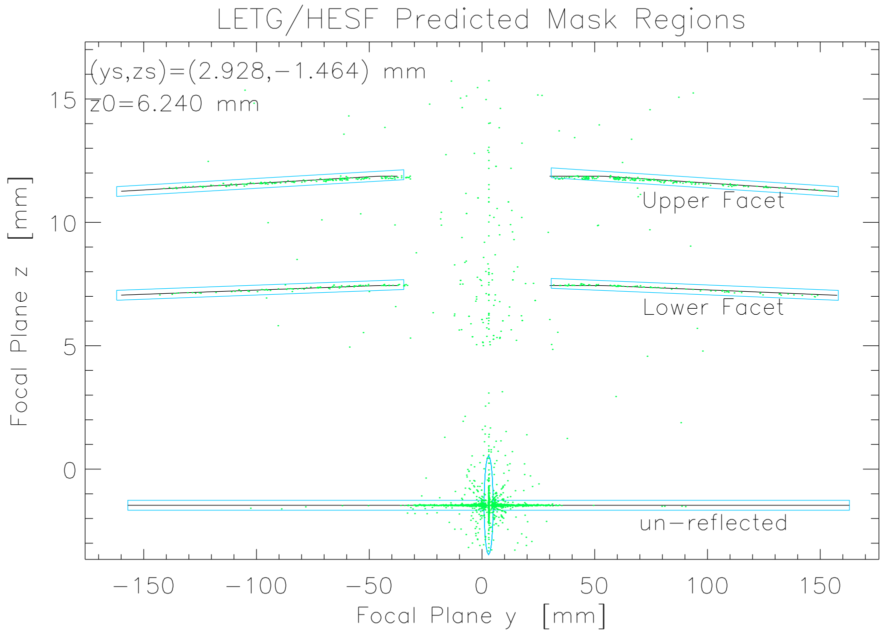
The following table lists the observational component to the A-coefficient for lower and upper facet reflections. (Units are in mm. The other coefficients can be found in the previous table on page 4; they don't change w/ source or SIM location.)

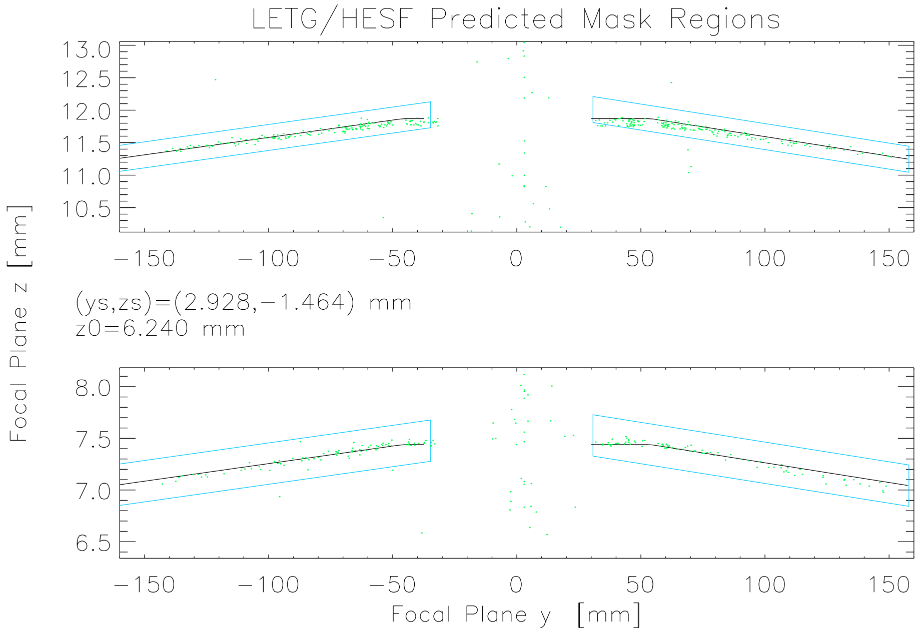
z_s	z_o	A_{ol}	A_{ou}
0.000	6.240	12.558	12.671
1.464	6.240	11.076	11.162
0.000	6.240	12.558	12.671
0.000	5.471	11.010	11.109
-0.878	5.471	11.900	12.015
0.878	6.250	11.689	11.786
0.000	6.250	12.578	12.691

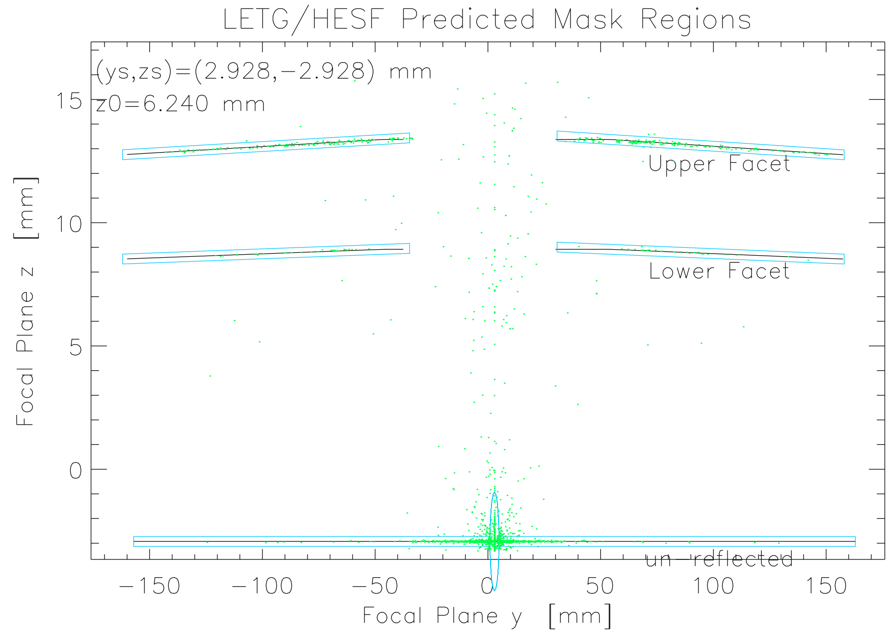
Table 3: Parameters and coefficients for different pointings and zero-order positions.

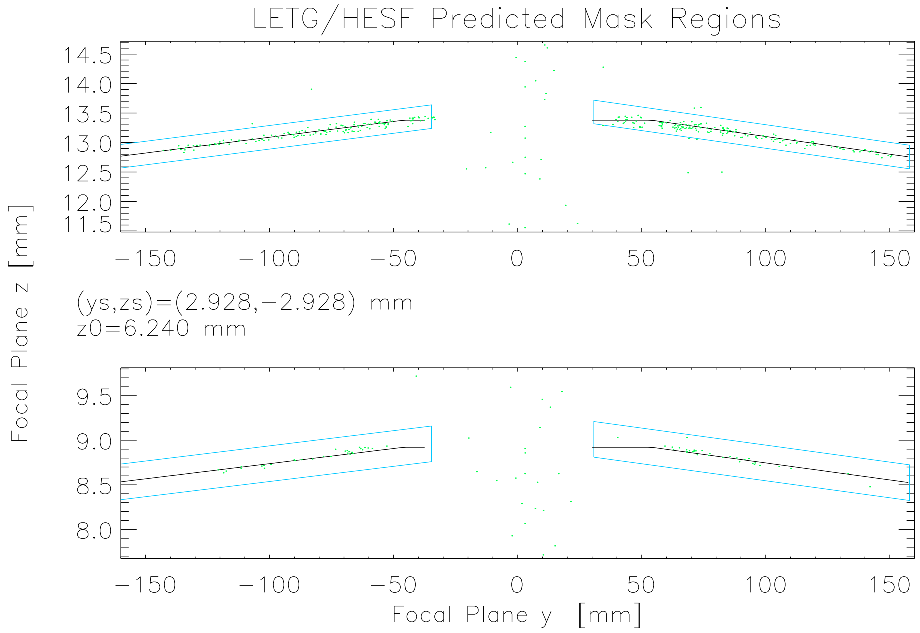
The following figures show the predicted spectrum and mask locations as well as a MARX simulation for the same cases. Each figure is labeled with the source and SIM positions.

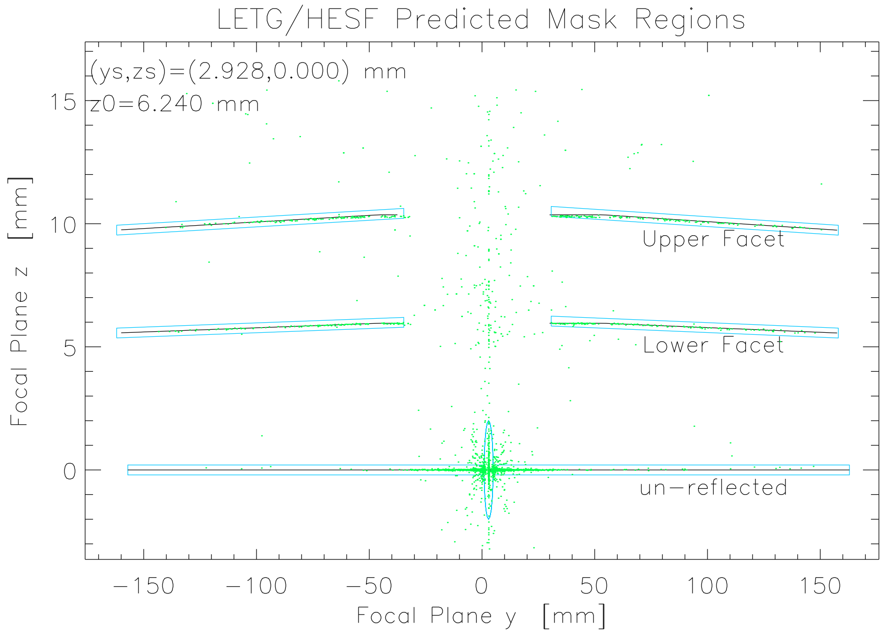

LETG/HESF Predicted Mask Regions
(ys,zs)=(0.000,0.000) mm
z0=6.240 mm
Upper Facet
Lower Facet
un−reflected
Focal Plane z [mm]
Focal Plane y [mm]
LETG/HESF Predicted Mask Regions
(ys,zs)=(0.000,0.000) mm
z0=6.240 mm
Focal Plane z [mm]
Focal Plane y [mm]

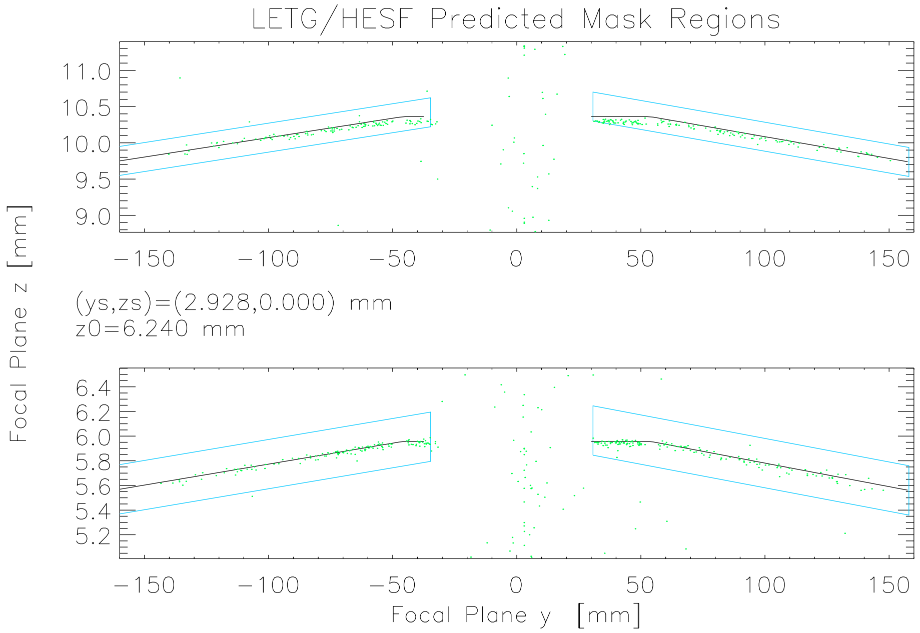

LETG/HESF Predicted Mask Regions
(ys,zs)=(2.928,−1.464) mm
z0=6.240 mm
Upper Facet
Lower Facet
un−reflected
Focal Plane z [mm]
Focal Plane y [mm]
15
10
5
0
−150
−100
−50
0
50
100
150

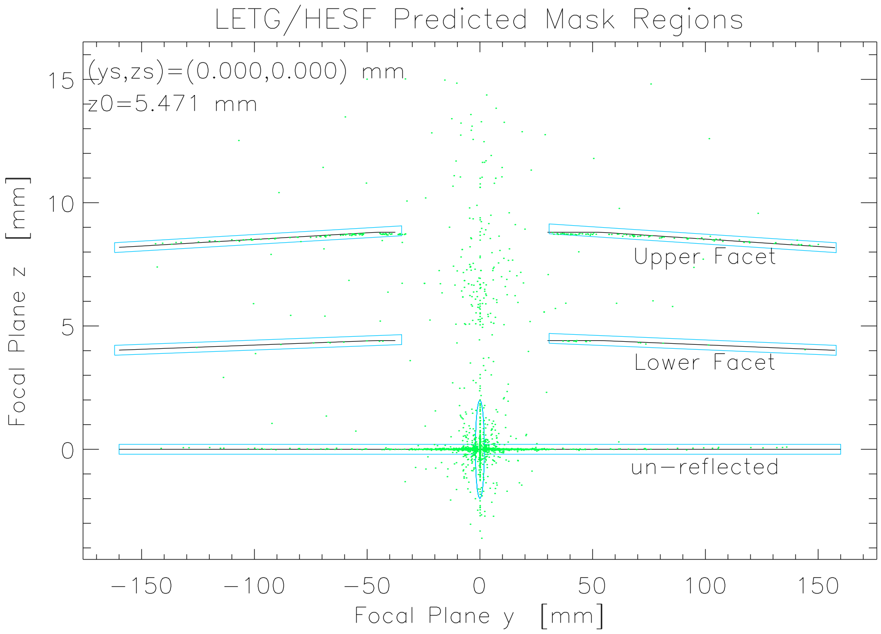

LETG/HESF Predicted Mask Regions
13.0
12.5
12.0
11.5
11.0
10.5
−150
−100
−50
0
50
100
150
(ys,zs)=(2.928,−1.464) mm
z0=6.240 mm
Focal Plane z [mm]
8.0
7.5
7.0
6.5
Focal Plane y [mm]

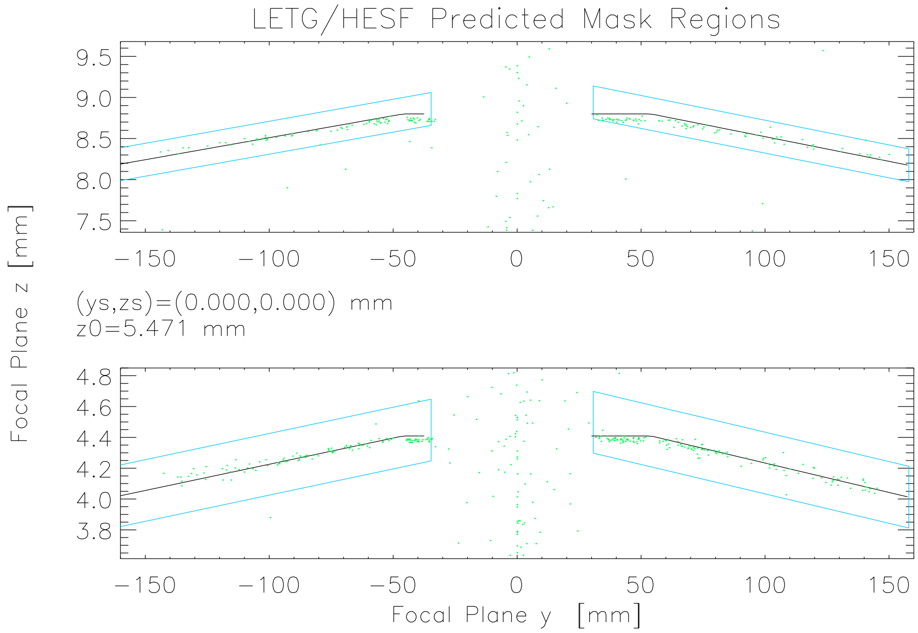

LETG/HESF Predicted Mask Regions
(ys,zs)=(2.928,−2.928) mm
z0=6.240 mm
Upper Facet
Lower Facet
un−reflected
Focal Plane z [mm]
Focal Plane y [mm]

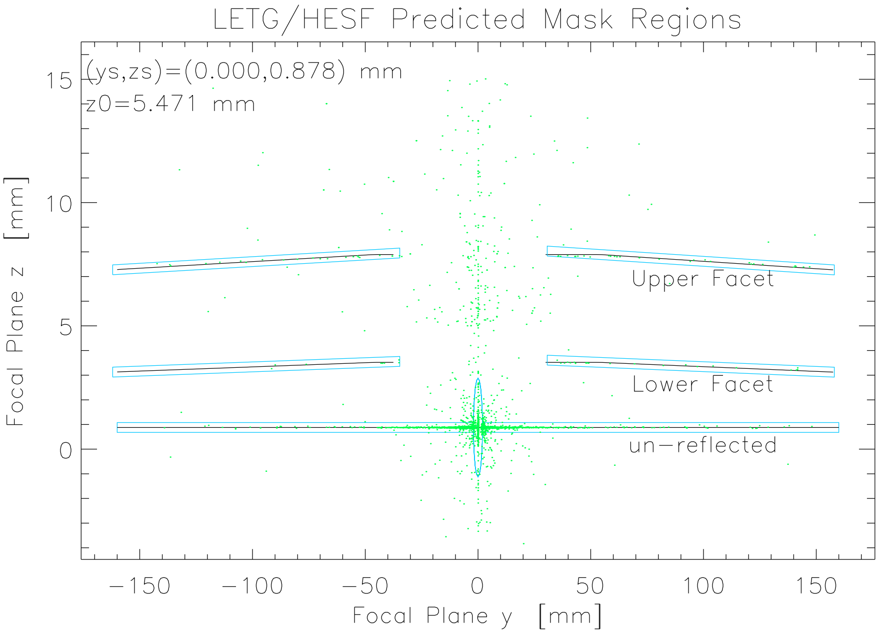

LETG/HESF Predicted Mask Regions
(ys,zs)=(2.928,−2.928) mm
z0=6.240 mm
Focal Plane z [mm]
Focal Plane y [mm]

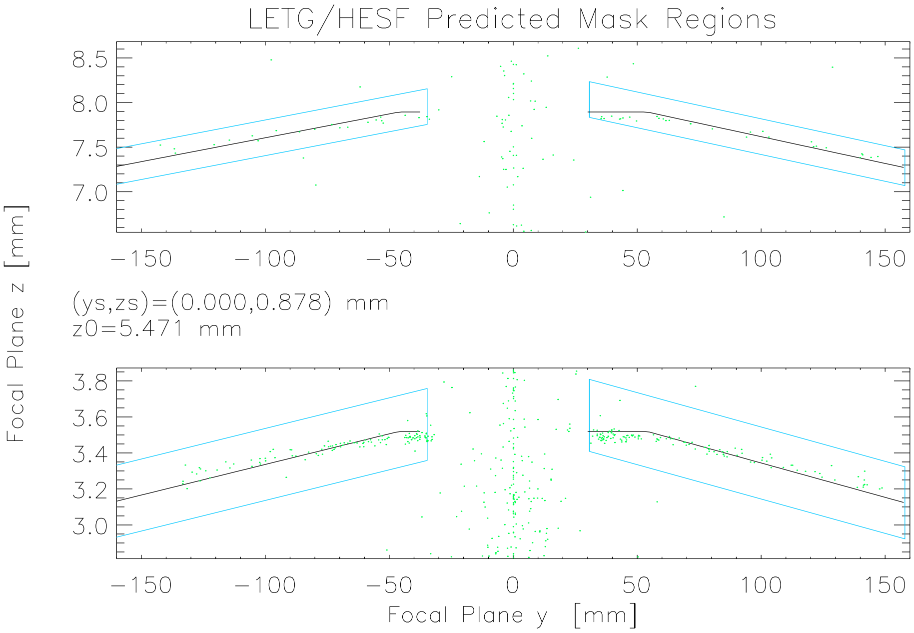

LETG/HESF Predicted Mask Regions
(ys,zs)=(2.928,0.000) mm
z0=6.240 mm
Upper Facet
Lower Facet
un−reflected
Focal Plane z [mm]
Focal Plane y [mm]

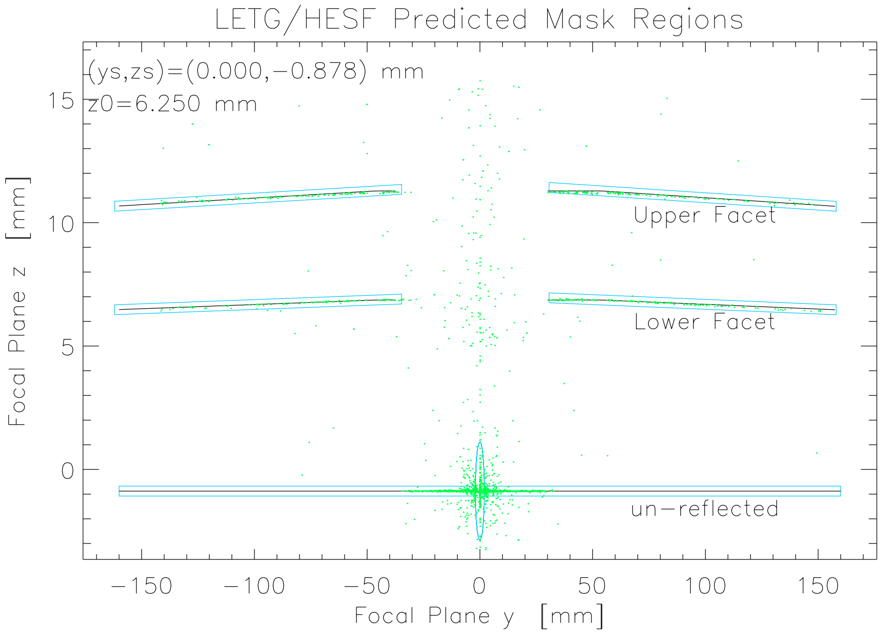

LETG/HESF Predicted Mask Regions
(ys,zs)=(2.928,0.000) mm
z0=6.240 mm
Focal Plane z [mm]
Focal Plane y [mm]

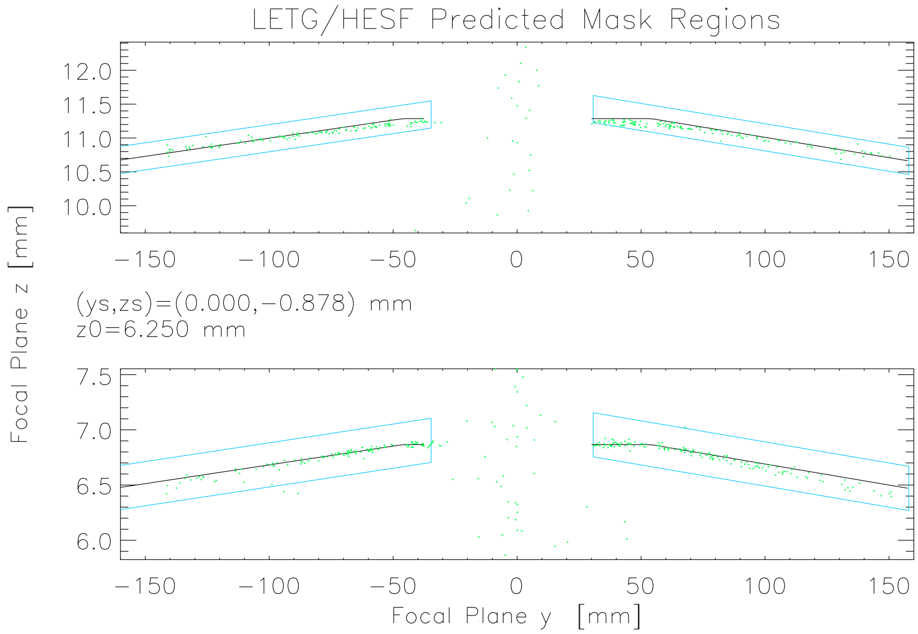

LETG/HESF Predicted Mask Regions
(ys,zs)=(0.000,0.000) mm
z0=5.471 mm
Upper Facet
Lower Facet
un−reflected
Focal Plane z [mm]
Focal Plane y [mm]

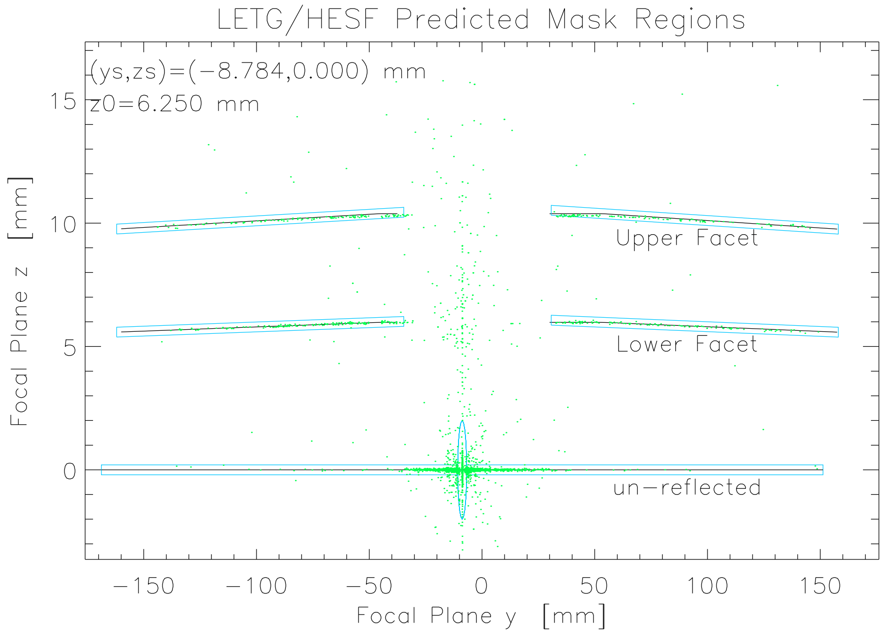

LETG/HESF Predicted Mask Regions
(ys,zs)=(0.000,0.000) mm
z0=5.471 mm
Focal Plane z [mm]
Focal Plane y [mm]

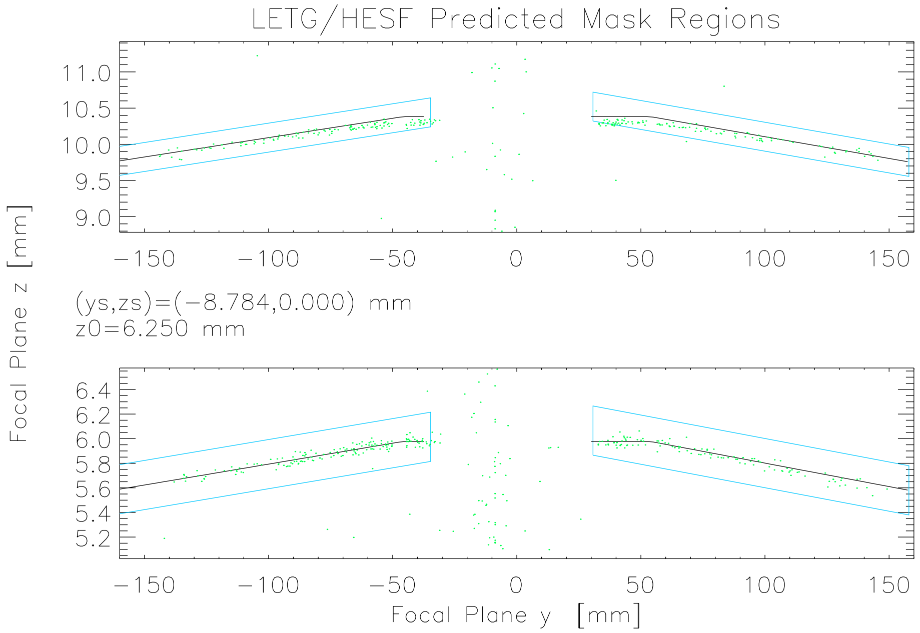

LETG/HESF Predicted Mask Regions
(ys,zs)=(0.000,0.878) mm
z0=5.471 mm
Upper Facet
Lower Facet
un−reflected
Focal Plane z [mm]
Focal Plane y [mm]
15
10
5
0
−150
−100
−50
0
50
100
150


LETG/HESF Predicted Mask Regions
8.5
8.0
7.5
7.0
−150
−100
−50
0
50
100
150
(ys,zs)=(0.000,0.878) mm
z0=5.471 mm
Focal Plane z [mm]
3.8
3.6
3.4
3.2
3.0
−150
−100
−50
0
50
100
150
Focal Plane y [mm]


LETG/HESF Predicted Mask Regions
(ys,zs)=(0.000,−0.878) mm
z0=6.250 mm
Upper Facet
Lower Facet
un−reflected
Focal Plane z [mm]
Focal Plane y [mm]


LETG/HESF Predicted Mask Regions
(ys,zs)=(0.000,−0.878) mm
z0=6.250 mm
Focal Plane z [mm]
Focal Plane y [mm]


LETG/HESF Predicted Mask Regions
(ys,zs)=(−8.784,0.000) mm
z0=6.250 mm
Upper Facet
Lower Facet
un−reflected
Focal Plane z [mm]
Focal Plane y [mm]


LETG/HESF Predicted Mask Regions
(ys,zs)=(−8.784,0.000) mm
z0=6.250 mm
Focal Plane z [mm]
Focal Plane y [mm]

B Specification Notes for Developers

This section provides additional detail required to implement the HESF mask generation. The computation is similar to that in the tool `tg_create_mask`. The region product is specified in the gratings Level 1.5 ICD. It will be used in `tg_resolve_events`, which currently contains some logic for classifying photons using the HESF mask definition, but not for deriving wavelengths.

One fundamental difference is that not only must the zero-order sky centroid be known, but also the average distance of the zero order from the HRC-S centerline; since the regions are reflected by the flat, their separations change with this coordinate.

A difference for wavlength calculation in `tg_resolve_events` is that the `CHIPY` will have to be altered temporarily prior to calculation of grating angular coordinates (GAC) and wavelength coordinates, so that `pixlib` may be called identically to non-HESF observations.

B.1 Mask Calculation Overview

`hesf_mask_calc` Calculate a region mask for an LETG/HESF observation. (not necessarily a separate tool; could be a mode to `tg_create_mask`).

- Inputs:

  zero-order centroid: Position of the zero-order in the aspect-corrected sky coordinate system; real-valued (from `tgdetect`).

  calibration parameters file File holding calibration parameters relevant to the HESF. (`HESF_par.fits`; see below)

- Parameters:

  zero-order width factor: a factor by which to expand the zero-order radius obtained from an error-budget calculation (see `tg_create_mask`).

  diffracted order width factor: a factor by which to expand the diffracted order widths obtained from an error-budget calculation (see `tg_create_mask`).

  observational parameters: Various observational parameters, as required (e.g., `SIM_Z`, RA_NOM, DEC_NOM; from "obs.par")

- Outputs:

  region file: HESF region file (ascii).

B.2 Mask Calculation Processing

Coordinate transformations are usually done with the `pixlib` routines. Where I use z in units of mm, there will be an equivalent `DETY`, and likewise for y, which corresponds to `DETX`. (`DETX`, `DETY`) is the un-aspect-corrected focal-plane detector coordinate system, which is similar to the sky (x, y) system in pixel size.

---

1. Determine z_0 for the observation, the z-distance from the optical axis (by definition at $z = 0$) to the HRC-S centerline. This is determined by the `SIM_Z` value from the observational parameters, and the calibration of the `SIM_Z` when the optical axis is on the centerline.

   This is nominally 6.25 mm for HESF observations, but is generally given by:

   $$z_0 = \texttt{SIM\_Z} - \texttt{SIM\_Z}_{\texttt{center}},$$

   where $\texttt{SIM\_Z}_{center}$ comes from calibration parameters (TBD).

   Check sign. aimpt-HS1 is 250.1, and is in pix_sim_table_flight.par

2. Determine z_s, the zero-order coordinate corresponding to the sky zero-order centroid. This is a mean or representative value, since aspect dither changes this with time. Any time will suffice, since the mask separations in z will be invariant over the scale of the dither. (Though the mask itself translates with dither, we only need to find the relative positions for one instant, since de-aspecting will occur in `tg_resolve_events` and also because the masks will be widened sufficiently to accomodate small inaccuracies.)

   z_s can be determined by "de-aspecting" the zero-order sky centroid to get the detector (or focal-plane) position.

3. Use z_0 and z_s to calculate the only quantity which is observation dependent:

   $$A_{oi} \approx 2(1 + \theta_i^2)z_0 - (1 + 2\theta_i^2)z_s. \tag{10}$$

   (Note: subscript o means "observed", and subscript i is for upper ("u") or lower ("l") segments of the HESF mirrors.)

   The parameters, θ_i, are obtained from the calibration parameters file, as values `HESF_THETA_UPPER` and `HESF_THETA_LOWER` in degrees.

   Hence, there are two observationally dependent values obtained: A_{ou} and A_{ol}.

   Nominal values are given in Tables 2 for parameters in Table 1 and for other pointings in Table 3.

4. Determine the mid-points between the gap inner edge and the array edge, on both the plus and minus sides. These won't change – they depend only on the calibration. These are:

   $$y_{0p} = \frac{1}{2}\left(\max(y) + y_{gp}\right);$$
   $$y_{0m} = \frac{1}{2}\left(\min(y) + y_{gm}\right).$$

   (Subscript "m" is for "minus", and "p" for "plus", referring to each side for diffracted orders.) The gap position parameters come from the calibration parameters file as: `HESF_GAPY_MINUS` (y_{gm}), `HESF_GAPY_PLUS` (y_{gp}). The minimum and maximum y are the limits of the HRC-S array in AXAF-y coordinates map to parameters called `HRCS_YMIN` and `HRCS_YMAX`. param names TBR

5. Calculate the "B" coefficients — total of four, two for each flat segment (upper, lower), and for each side (plus, minus).

$$B_{um} = -\tan 2\theta_u \, \tan\delta_m$$
$$B_{lm} = -\tan 2\theta_l \, \tan\delta_m$$
$$B_{up} = -\tan 2\theta_u \, \tan\delta_p$$
$$B_{lp} = -\tan 2\theta_l \, \tan\delta_p$$

   See Table 4 for a mapping of terms to parameters in the calibration parameter file.

6. Calculate four "calibration-A" coefficients (each segment, and each side):

$$A_{cum} = \tan 2\theta_u \left[h_u \left(1 - \frac{\tan\theta_l}{\tan\theta_u}\right) - \left(\frac{z_f}{\tan\theta_u}\right) + h_l(0) + y_{0m}\tan\delta_m \right]$$
$$A_{clm} = \tan 2\theta_l \left[-\left(\frac{z_f}{\tan\theta_l}\right) + h_l(0) + y_{0m}\tan\delta_m \right]$$
$$A_{cup} = \tan 2\theta_u \left[h_u \left(1 - \frac{\tan\theta_l}{\tan\theta_u}\right) - \left(\frac{z_f}{\tan\theta_u}\right) + h_l(0) + y_{0p}\tan\delta_p \right]$$
$$A_{clp} = \tan 2\theta_l \left[-\left(\frac{z_f}{\tan\theta_l}\right) + h_l(0) + y_{0p}\tan\delta_p \right]$$

7. Add the "calibration" and "observational" A-coefficients:

$$A_{um} = A_{ou} + A_{cum},$$
$$A_{up} = A_{ou} + A_{cup},$$
$$A_{lm} = A_{ol} + A_{clm},$$
$$A_{lp} = A_{ol} + A_{clp}.$$

8. Calculate the values of z at the values of y for the upper and lower facets:

$$z_{um} = A_{um} + B_{um} y_{0m},$$
$$z_{up} = A_{up} + B_{up} y_{0p},$$
$$z_{lm} = A_{lm} + B_{lm} y_{0m}.$$
$$z_{lp} = A_{lp} + B_{lp} y_{0p},$$

9. Define the centers of four rectangular regions with these coordinates:

$$(y_{0m}, z_{um}),$$
$$(y_{0p}, z_{up}),$$
$$(y_{0m}, z_{lm}),$$
$$(y_{0p}, z_{lp}).$$

10. Assign a width to the rectangular region. Widths are specified via error-budget calculation as in non-HESF Level 1.5 processing (incorporating PSF vs off-axis angle lookup and defocus, astigmatism, and an arbitrary multiplication factor); typical widths are about ± 0.2 mm.

11. Assign a length to the rectangular region. The length need not be precise. It should extend in the y-direction from the central HESF gap to the detector edge, or about
$$L_p \approx \max(y) + y_{gp};$$
$$L_m \approx -(\min(y) + y_{gm}).$$

12. Determine the rotation of each rectangle. These do not depend upon any observational parameters, but only on calibration parameters. The rotations are given by the B_i values:
$$\rho_{up} = \arctan(B_{up}),$$
$$\rho_{um} = \arctan(B_{um}),$$
$$\rho_{lp} = \arctan(B_{lp}),$$
$$\rho_{lm} = \arctan(B_{lm}).$$

13. Transform the centers, widths, lengths, and rotations from focal-plane to the sky frame.

B.3 Variable-to-Parameter Map

The variables used in the equations and given in Table 1 are expanded here into each component ("minus" or "plus" sides, or "upper" or "lower" segment of the flat), and matched to names in the calibration parameter file. The names have been chosen to be identical to `MARX (v2.20)` parameter names, when there is a direct correspondence. These parameters are stored in "Analysis Reference Data" (ARD) files, and are documented in the ICD, "Analysis Reference Data for: Exposure Maps and Grating Levels 1.5-2 Pipelines (Revision 1.4)" (see the section entitled "HRC-S Parameters"[6]

Term	**Parameter Name**	**Description**
z_f	HESFOffsetZ	The distance in the z-axis between the center of the HRC-S and the lower edge of the filter.[7]
θ_l	HESFTheta1	the angle in the $x - z$ plane between the optical axis (x) and the plane of the upper facet.
θ_u	HESFTheta2	The angle in the $x - z$ plane between the optical axis (x) and the plane of the lower facet.
h_u	HESFHeight1	The x axis distance between the lower edge of the upper facet (nearest the HRMA) and the lower edge of the lower facet.
$h_l(0)$	HESFOffsetX	The x axis distance (or "height") of the lower edge of the lower facet (furthest from the HRMA) above the MCP surface, at $y = 0$.
$y_0(y)$	HRCS_Y0_MINUS HRCS_Y0_PLUS	The "boundaries" of MCP segments, relative to optical axis (which is different from the array centerline). These mark the locations at which the MCP tilts begin and $h_l(y) \neq h_l(0)$.
$\delta(y)$	HRCS_TILT_MINUS HRCS_TILT_CENTER HRCS_TILT_PLUS	The tilt angle of the MCP (which which puts the y into $h_l(y)$). The negative is a convention to get the proper signs on the negative-y side. Note the y-limits should be the same as the values of $y_0(y)$.
y_{gm}	HESFGapY2	The HESF central gap limit on the minus-y side.
y_{gp}	HESFGapY1	The HESF central gap limit on the plus-y side.
$\min(y)$	HRCS_YMIN	The minimum extent of the HRC-S on the AXAF y-axis.
$\max(y)$	HRCS_YMAX	The maximum extent of the HRC-S on the AXAF y-axis.

Table 4: Variable to Parameter Mapping. Parameter names are identical to `MARX` parameter names if there is a direct correspondence. These parameters are stored in a FITS parameter Analysis Reference Data file containing HRC-S paramters.

[6] Drafts can be found at `http://space.mit.edu/ASC/docs/ARD_ICD/ARD_ICD.ps.gz` and approved versions on the ICD table at `http://hea-www.harvard.edu/asclocal/sds/development/ICD/index.html`

B.4 Grating Coordinate Determination

In order to calculate photon grating angular coordinates and wavelengths without modifications to `pixlib`, we can temporarily modify the z-coordinate (`DETY`, or `CHIPY`) of photons in the HESF mask regions. To do so, we will determine how far each photon is from the nominal centerline of the HESF regions; for example, in Figure 2, this line is drawn in the center of each rotated rectangle surrounding the events' positions. We then put the photon that same distance from the un-reflected region centerline, given by the instantaneous zero-order z-coordinate (`DETY`). This can be written as:

$$z' = (A_i + B_i y) - z + z_s.$$

We will, however, need to compute the A and B values for the "kinked" region near the center of the field (see Figure 2). These were ignored when making the masks to simplify the regions (four rectangles instead of eight), since the rectanglar masks are made wide enough to enclose the kink. To make the terms explicit, we will add to the subscripts "l" and "u" for "lower" and "upper" flat segments, and "m" and "p" for "minus" and "plus" order sides, the subscript "μ" (for "middle").

Note also that the observational component, A_{oi}, now depends upon time through z_s, and will have to be computed for each photon time.

1. Calucate A_c and B coefficients as for mask-generation. (See item 5 on page 18.)

2. Calculate "B"-coefficients for the middle section:

$$B_{u\mu} = -\tan 2\theta_u \, \tan \delta_\mu$$

$$B_{l\mu} = -\tan 2\theta_l \, \tan \delta_\mu$$

(These are zero for nominal parameters).

3. Calculate "calibration A"-coefficients for the middle section:

$$A_{cu\mu} = \tan 2\theta_u \left[h_u \left(1 - \frac{\tan\theta_l}{\tan\theta_u} \right) - \left(\frac{z_f}{\tan\theta_u} \right) + h_l(0) + y_{0\mu} \tan\delta_\mu \right]$$

$$A_{cl\mu} = \tan 2\theta_l \left[- \left(\frac{z_f}{\tan\theta_l} \right) + h_l(0) + y_{0\mu} \tan\delta_\mu \right]$$

As designed, $\delta_\mu = 0.0$, so these become:

$$A_{cu\mu} = \tan 2\theta_u \left[h_u \left(1 - \frac{\tan\theta_l}{\tan\theta_u} \right) - \left(\frac{z_f}{\tan\theta_u} \right) + h_l(0) \right]$$

$$A_{cl\mu} = \tan 2\theta_l \left[- \left(\frac{z_f}{\tan\theta_l} \right) + h_l(0) \right]$$

4. Determine z_0 for the observation, the z-distance from the optical axis (by definition at $z = 0$) to the HRC-S centerline. (See item 1 on page 17.)

5. For each photon in each region:

(a) Determine z_s, the z coordinate corresponding to the zero-order sky centroid at the event's time (via an aspect inversion).

(b) Compute the "observed-A"-coefficient appropriate to the photon's mask component ("l" or "u", and "m", "μ", or "p"):

$$A_{oum} = 2(1+\theta_u^2)z_0 - (1+2\theta_u^2)z_s,$$
$$A_{ou\mu} = 2(1+\theta_u^2)z_0 - (1+2\theta_u^2)z_s,$$
$$A_{oup} = 2(1+\theta_u^2)z_0 - (1+2\theta_u^2)z_s,$$
$$A_{olm} = 2(1+\theta_l^2)z_0 - (1+2\theta_l^2)z_s,$$
$$A_{ol\mu} = 2(1+\theta_l^2)z_0 - (1+2\theta_l^2)z_s,$$
$$A_{olp} = 2(1+\theta_l^2)z_0 - (1+2\theta_l^2)z_s.$$

The selection of μ is made if the event has a y such that

$$y_{0m} < y \leq y_{0p}.$$

Or, as a simpler test, the event's `CHIP_ID` $= 2$ (the central HRC-S MCP). And likewise, "m" corresponds to $\mathtt{CHIP_ID} = 1$, and "p" to `CHIP_ID` $= 3$.

(c) Compute the total A-coefficient (appropriate for the photon region and MCP as in the previous step):

$$A_{um} = A_{cum} + Aoum,$$
$$A_{u\mu} = A_{cu\mu} + Aou\mu,$$
$$A_{up} = A_{cup} + Aoup,$$
$$A_{lm} = A_{clm} + Aolm,$$
$$A_{l\mu} = A_{cl\mu} + Aol\mu,$$
$$A_{lp} = A_{clp} + Aolp.$$

(d) Compute the modified z-coordinate of the photon:

$$z' = (A + By) - z + z_s.$$

(e) Compute diffraction coordinates (GDP, GAC, wavelength) for this photon at this time, as usual (via `pixlib` call).