MIT Kavli Institute **Chandra X-Ray Center**

MEMORANDUM
December 9, 2016

To: Jonathan McDowell, SDS Group Leader
From: Glenn E. Allen, SDS
Subject: `mkacisrmf` spec
Revision: 1.00
URL: http://space.mit.edu/CXC/docs/docs.html#mkacisrmf
File: /nfs/inconceivable/d0/sds/specs/mkacisrmf/mkacisrmf_spec_1.00.tex

1 Description

This spec, which is incomplete, describes how **`mkacisrmf`** computes an ACIS PHA RMF for a single region.

2 Parameters

1. `infile`,f,a,“ ”,,,“Input CALDB file”
2. `energy`,s,a,” “,,,”Energy grid in keV”
3. `channel`,s,a,” “,,,”Channel grid”

3 Error checking

1. `infile`:
   (a) Existence:
   If the `infile` does not exist, then **`mkacisrmf`** exits with an error message.
   (b) Permission:
   If the `infile` exists, but the file permissions do not allow it to be read, then **`mkacisrmf`** exits with an error message.
   (c) Ideal front-illuminated RMF:
      i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \text{CDB\_ACIS\_RESPONSE} \tag{1}$$

      that is appropriate for the front-illuminated CCDs, then **`mkacisrmf`** exits with an error message. Hereafter, this HDU is refered to as $h_{\mathrm{ideal,fi}}$.

ii. If the `infile` contains an HDU $h_{\mathrm{ideal,fi}}$, but the binary table does not include the columns `ENERGY`, `NGRP`, `PHABOUND`, `AVERESP`, and `SHIFT`, then `mkacisrmf` exits with an error message.

iii. If the `infile` contains an HDU $h_{\mathrm{ideal,fi}}$, but the header does not include the keywords `GAIN0`, `GAIN00`, `GAIN01`, `GAIN02`, `GAIN03`, `GAIN10`, `GAIN11`, `GAIN12`, `GAIN13`, `GAIN20`, `GAIN21`, `GAIN22`, `GAIN23`, `GAIN30`, `GAIN31`, `GAIN32`, `GAIN33`, `GAIN40`, `GAIN41`, `GAIN42`, `GAIN43`, `GAIN60`, `GAIN61`, `GAIN62`, `GAIN63`, `GAIN80`, `GAIN81`, `GAIN82`, `GAIN83`, `GAIN90`, `GAIN91`, `GAIN92`, `GAIN93`, `SHIFT00`, `SHIFT01`, `SHIFT02`, `SHIFT03`, `SHIFT10`, `SHIFT11`, `SHIFT12`, `SHIFT13`, `SHIFT20`, `SHIFT21`, `SHIFT22`, `SHIFT23`, `SHIFT30`, `SHIFT31`, `SHIFT32`, `SHIFT33`, `SHIFT40`, `SHIFT41`, `SHIFT42`, `SHIFT43`, `SHIFT60`, `SHIFT61`, `SHIFT62`, `SHIFT63`, `SHIFT80`, `SHIFT81`, `SHIFT82`, `SHIFT83`, `SHIFT90`, `SHIFT91`, `SHIFT92`, and `SHIFT93`, then `mkacisrmf` exits with an error message.

(d) Ideal back-illuminated RMF:

i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \mathrm{CDB\_ACIS\_RESPONSE} \qquad (2)$$

that is appropriate for the back-illuminated CCDs, then `mkacisrmf` exits with an error message. Hereafter, this HDU is refered to as $h_{\mathrm{ideal,bi}}$.[1]

ii. If the `infile` contains an HDU $h_{\mathrm{ideal,bi}}$, but the binary table does not include the columns `ENERGY`, `NGRP`, `PHABOUND`, `AVERESP`, and `SHIFT`, then `mkacisrmf` exits with an error message.

iii. If the `infile` contains an HDU $h_{\mathrm{ideal,bi}}$, but the header does not include the keywords `GAIN0`, `GAIN50`, `GAIN51`, `GAIN52`, `GAIN53`, `GAIN70`, `GAIN71`, `GAIN72`, `GAIN73`, `SHIFT50`, `SHIFT51`, `SHIFT52`, `SHIFT53`, `SHIFT70`, `SHIFT71`, `SHIFT72`, and `SHIFT73`, then `mkacisrmf` exits with an error message.

(e) Front-illuminated scatter matrix data:

i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \mathrm{CDB\_ACIS\_RESP\_CTI} \qquad (3)$$

that is appropriate for the front-illuminated CCDs, then `mkacisrmf` exits with an error message. Hereafter, this HDU is refered to as $h_{\mathrm{scatter,fi}}$.

ii. If the `infile` contains an HDU $h_{\mathrm{scatter,fi}}$, but the binary table does not include the columns `CCD_ID`, `CHIPX_LO`, `CHIPX_HI`, `CHIPY_LO`, `CHIPY_HI`, `PHACHAN`, `L1_WIDTH`, and `L1_POS`, then `mkacisrmf` exits with an error message.

iii. If the `infile` contains an HDU $h_{\mathrm{scatter,fi}}$, but the header does not include the keywords `L1ALPH1` and `L1ALPH2`, then `mkacisrmf` exits with an error message.

(f) Back-illuminated scatter matrix data:

i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \mathrm{CDB\_ACIS\_RESP\_CTI} \qquad (4)$$

that is appropriate for the back-illuminated CCDs, then `mkacisrmf` exits with an error message. Hereafter, this HDU is refered to as $h_{\mathrm{scatter,bi}}$.[2]

ii. If the `infile` contains an HDU $h_{\mathrm{scatter,bi}}$, but the binary table does not include the columns `CCD_ID`, `CHIPX_LO`, `CHIPX_HI`, `CHIPY_LO`, `CHIPY_HI`, `PHACHAN`, `G1_FWHM`, `G1_POS`, `G1_AMPL`, `G2_FWHM`, `G2_POS`, and `G2_AMPL`, then `mkacisrmf` exits with an error message.

(g) Gain tweak data:

[1] Hereafter h_{ideal} is used to generically refer to $h_{\mathrm{ideal,fi}}$ for a front-illuminated CCD or $h_{\mathrm{ideal,bi}}$ for a back-illuminated CCD.

[2] Hereafter h_{scatter} is used to generically refer to $h_{\mathrm{scatter,fi}}$ for a front-illuminated CCD or $h_{\mathrm{scatter,bi}}$ for a back-illuminated CCD.

i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \text{CDB\_ACIS\_RESP\_GCORR}, \tag{5}$$

then `mkacisrmf` exits with an error message. Hereafter, this HDU is refered to as h_{gtweak}.

ii. If the `infile` contains an HDU h_{gtweak}, but the binary table does not include the columns `ENERGY`, `GTWEAK0`, `GTWEAK1`, `GTWEAK2`, `GTWEAK3`, `GTWEAK4`, `GTWEAK5`, `GTWEAK6`, `GTWEAK7`, `GTWEAK8`, and `GTWEAK9`, then `mkacisrmf` exits with an error message.

2. `energy`:

   (a) Number:
   If N_{energy}, the number of elements in the input `energy` grid, is less than two, then `mkacisrmf` exits with an error message.

   (b) Minimum:
   If `energy`[0], the first element in the `energy` grid, is less than `ENERGY`[0], where `ENERGY`[0] is the value in the first row of the column `ENERGY` of HDU h_{ideal}, then `mkacisrmf` exits with an error message.

   (c) Maximum:
   If `energy`$[N_{\text{energy}} - 1]$, the last element in the `energy` grid, is greater than `ENERGY`$[N_{\text{ideal}} - 1]$, where `ENERGY`$[N_{\text{ideal}} - 1]$ is the value in the last row of the column `ENERGY` of HDU h_{ideal}, then `mkacisrmf` exits with an error message.

3. `channel`:

   (a) Number:
   If N_{channel}, the number of elements in the input `channel` grid, is less than two, then `mkacisrmf` exits with an error message.

   (b) Minimum:
   If `channel`[0], the first element in the `channel` grid, is less than 1, then `mkacisrmf` exits with an error message.

   (c) Maximum:
   If `channel`$[N_{\text{channel}}-1]$, the last element in the `channel` grid, is greater than 4096, then `mkacisrmf` exits with an error message.

4 Processing

An RMF is a two-dimensional array $R[i, j]$ with $N_{\text{energy}} - 1$ energy elements i and N_{channel} elements j. Here, $N_{\text{channel}} = 4096$. The three basic steps to computing an RMF, which are described hereafter, are: (1) obtaining a "position-independent,[3]" ideal RMF, (2) obtaining a position-dependent scatter matrix, and (3) convolving the ideal RMF and scatter matrix.

The steps in sections 4.1–4.3 are performed for each `energy` index $i = 0, 1, \ldots, N_{\text{energy}} - 2$.

4.1 Ideal RMF

To obtain the "ideal" or "pre-CTI," response for `energy` bin i:

1. The mean `energy` is computed:

$$E_{\text{mean}}[i] = \frac{\texttt{energy}[i] + \texttt{energy}[i+1]}{2}. \tag{6}$$

[3] There is one ideal front-illuminated RMF and one ideal back-illuminated RMF.

2. The mean `energy` is tweaked:

   (a) If

   $$E_{\mathrm{mean}}[i] \quad < \quad \mathtt{ENERGY}[0], \tag{7}$$

   where $\mathtt{ENERGY}[0]$ is the value in the first row of the column `ENERGY` in HDU h_{gtweak}, then

   $$E_{\mathrm{tweaked}}[i] \quad = \quad E_{\mathrm{mean}}[i]\,\mathtt{GTWEAK}c[0], \tag{8}$$

   where $c = 0, 1, \ldots,$ or 9, depending on the value of the `CCD_ID` of the region, and $\mathtt{GTWEAK}c$ is a column in HDU h_{gtweak}.

   (b) If

   $$E_{\mathrm{mean}}[i] \quad \geq \quad \mathtt{ENERGY}[N_{\mathrm{gtweak}} - 1], \tag{9}$$

   where N_{gtweak} is the number of rows in the column $\mathtt{GTWEAK}c$, then

   $$E_{\mathrm{tweaked}}[i] \quad = \quad E_{\mathrm{mean}}[i]\,\mathtt{GTWEAK}c[N_{\mathrm{gtweak}} - 1]. \tag{10}$$

   (c) If there is a row r such that

   $$E_{\mathrm{mean}}[i] \quad \geq \quad \mathtt{ENERGY}[r] \text{ and} \tag{11}$$
   $$E_{\mathrm{mean}}[i] \quad < \quad \mathtt{ENERGY}[r+1], \tag{12}$$

   then

   $$E_{\mathrm{tweaked}}[i] \quad = \quad E_{\mathrm{mean}}[i]\left(\left[\frac{E_{\mathrm{mean}}[i] - \mathtt{ENERGY}[r]}{\mathtt{ENERGY}[r+1] - \mathtt{ENERGY}[r]}\right]\Big(\mathtt{GTWEAK}c[r+1] - \mathtt{GTWEAK}c[r]\Big) + \tag{13}$$
   $$\mathtt{GTWEAK}c[r]\right). \tag{14}$$

3. The ideal response is computed:

   (a) The row of HDU h_{ideal} corresponding to $E_{\mathrm{tweaked}}[i]$ is found:

      i. If

      $$E_{\mathrm{tweaked}}[i] \quad < \quad \mathtt{ENERGY}[0], \tag{15}$$

      where $\mathtt{ENERGY}[0]$ is the value in the first row of the column `ENERGY` of HDU h_{ideal}, then the row

      $$r \quad = \quad 0 \tag{16}$$

      and the weight for this row

      $$w \quad = \quad 1. \tag{17}$$

      ii. If

      $$E_{\mathrm{tweaked}}[i] \quad \geq \quad \mathtt{ENERGY}[N_{\mathrm{ideal}} - 1], \tag{18}$$

      then

      $$r \quad = \quad N_{\mathrm{ideal}} - 2 \text{ and} \tag{19}$$
      $$w \quad = \quad 0. \tag{20}$$

iii. If

$$E_{\text{tweaked}}[i] \geq \texttt{ENERGY}[0] \text{ and} \tag{21}$$
$$E_{\text{tweaked}}[i] < \texttt{ENERGY}[N_{\text{ideal}} - 1], \tag{22}$$

then r is the row such that

$$E_{\text{tweaked}}[i] \geq \texttt{ENERGY}[r] \text{ and} \tag{23}$$
$$E_{\text{tweaked}}[i] < \texttt{ENERGY}[r + 1] \tag{24}$$

and the weight

$$w = \frac{\texttt{ENERGY}[r+1] - E_{\text{tweaked}}[i]}{\texttt{ENERGY}[r+1] - \texttt{ENERGY}[r]}. \tag{25}$$

Items 4.1.3(a)i and 4.1.3(a)ii limit the energy range over which it is possible to create an RMF.[4]

(b) The mean PHA for each group in this row is computed:
For $m = 0, 1, \ldots, \texttt{NGRP}[r] - 2$, where $\texttt{NGRP}[r]$ is the value in row r of the column NGRP of HDU h_{ideal}:

i. Lower limit on PHA for group m:

$$j_{\text{lo}}[m] = \text{int}\left(\texttt{PHABOUND}[r, m]\left(\frac{\texttt{GAIN0}}{\texttt{GAIN}cn}\right) + \texttt{SHIFT}cn\right), \tag{26}$$

where $\texttt{PHABOUND}[r, m]$ is the m^{th} element of row r of the column PHABOUND of HDU h_{ideal} and GAIN0, GAINcn, and SHIFTcn are keywords in the same HDU. Again, $c = 0, 1, \ldots,$ or 9 and $n = 0, 1, 2,$ or 3, depending on the values of the CCD_ID and NODE_ID[5] of the region, respectively. If

$$j_{\text{lo}}[m] < 1, \tag{27}$$

then

$$j_{\text{lo}}[m] = 1. \tag{28}$$

ii. Upper limit on PHA for group m:

$$j_{\text{hi}}[m] = \text{int}\left(\left(\texttt{PHABOUND}[r, m+1] - 1\right)\left(\frac{\texttt{GAIN0}}{\texttt{GAIN}cn}\right) + \texttt{SHIFT}cn\right). \tag{29}$$

If

$$j_{\text{hi}}[m] > 4096, \tag{30}$$

then

$$j_{\text{hi}}[m] = 4096. \tag{31}$$

iii. Mean PHA for group m:

$$j_{\text{mean}}[m] = \left(\frac{\texttt{PHABOUND}[r, m] + \texttt{PHABOUND}[r, m+1] - 1}{2}\right)\left(\frac{\texttt{GAIN0}}{\texttt{GAIN}cn}\right) + \texttt{SHIFT}cn. \tag{32}$$

[4]For the CALDB file acisD2000-01-29p2_respN0006.fits, this range is 0.243–12.0 keV.
[5]The NODE_ID = 0, 1, 2, and 3 for CHIPX = 1–256, 257–512, 513–768, and 769–1024, respectively.

(c) The ideal response for this row is computed:
For $m = 0, 1, \ldots, \texttt{NGRP}[r] - 2$:

i. If

$$
\begin{aligned}
m &= 0 \text{ or} && (33)\\
m &= \texttt{NGRP}[r] - 2 \text{ or} && (34)\\
j_{\mathrm{lo}}[m] &= j_{\mathrm{hi}}[m], && (35)
\end{aligned}
$$

then, for $j = j_{\mathrm{lo}}[m], j_{\mathrm{lo}}[m] + 1, \ldots, j_{\mathrm{hi}}[m]$,

$$
\mathcal{R}_r[j-1] = \texttt{AVERESP}[r, m], \tag{36}
$$

where $\texttt{AVERESP}[r, m]$ is the m^{th} element of the column AVERESP in row r of HDU h_{ideal}.

ii. If

$$
\begin{aligned}
m &> 0 \text{ and} && (37)\\
m &< \texttt{NGRP}[r] - 2 \text{ and} && (38)\\
j_{\mathrm{lo}}[m] &< j_{\mathrm{hi}}[m], && (39)
\end{aligned}
$$

then, for $j = j_{\mathrm{lo}}[m], j_{\mathrm{lo}}[m] + 1, \ldots, \mathrm{int}\big((j_{\mathrm{lo}}[m] + j_{\mathrm{hi}}[m])/2\big) - 1$,

$$
\begin{aligned}
\mathcal{R}_r[j-1] &= \left(\frac{j - j_{\mathrm{mean}}[m-1]}{j_{\mathrm{mean}}[m] - j_{\mathrm{mean}}[m-1]}\right)\Big(\texttt{AVERESP}[r, m] - \texttt{AVERESP}[r, m-1]\Big) + && (40)\\
&\quad \texttt{AVERESP}[r, m-1] && (41)
\end{aligned}
$$

and for $j = \mathrm{int}\left((j_{\mathrm{lo}}[m] + j_{\mathrm{hi}}[m])/2\right), \mathrm{int}\left((j_{\mathrm{lo}}[m] + j_{\mathrm{hi}}[m])/2\right) + 1, \ldots, j_{\mathrm{hi}}[m]$,

$$
\begin{aligned}
\mathcal{R}_r[j-1] &= \left(\frac{j - j_{\mathrm{mean}}[m]}{j_{\mathrm{mean}}[m+1] - j_{\mathrm{mean}}[m]}\right)\Big(\texttt{AVERESP}[r, m+1] - \texttt{AVERESP}[r, m]\Big) + && (42)\\
&\quad \texttt{AVERESP}[r, m]. && (43)
\end{aligned}
$$

(d) The ideal response for this row is normalized:

i.

$$
\mathcal{R}_{r,\mathrm{tot}} = 0. \tag{44}
$$

ii. For $j = j_{\mathrm{lo}}[0], j_{\mathrm{lo}}[0] + 1, \ldots, j_{\mathrm{hi}}[\texttt{NGRP}[r] - 2]$,

$$
\mathcal{R}_{r,\mathrm{tot}} = \mathcal{R}_{r,\mathrm{tot}} + \mathcal{R}_r[j-1]. \tag{45}
$$

iii. For $j = j_{\mathrm{lo}}[0], j_{\mathrm{lo}}[0] + 1, \ldots, j_{\mathrm{hi}}[\texttt{NGRP}[r] - 2]$,

$$
\mathcal{R}_r[j-1] = \frac{\mathcal{R}_r[j-1]}{\mathcal{R}_{r,\mathrm{tot}}}. \tag{46}
$$

(e) The ideal response for the next row is computed:
Steps 4.1.3b–4.1.3d are repeated to compute $\mathcal{R}_{r+1}$ for row $r + 1$.

4. The pulse height shift is computed:
For $m = 0, 1, \ldots, \texttt{NGRP}[r] - 2$,

(a) For $j = \texttt{PHABOUND}[r, m], \texttt{PHABOUND}[r, m] + 1, \ldots, \texttt{PHABOUND}[r, m+1]$,

$$
\begin{aligned}
s[j-1] &= 0.01\left(\frac{\texttt{GAIN0}}{\texttt{GAIN}cn}\right)\Bigg[\left(\frac{\texttt{PHABOUND}[r, m+1] - j}{\texttt{PHABOUND}[r, m+1] - \texttt{PHABOUND}[r, m]}\right)\texttt{SHIFT}[r, m] + \\
&\quad \left(\frac{j - \texttt{PHABOUND}[r, m]}{\texttt{PHABOUND}[r, m+1] - \texttt{PHABOUND}[r, m]}\right)\texttt{SHIFT}[r, m+1]\Bigg], && (47)
\end{aligned}
$$

where $\texttt{SHIFT}[r, m]$ is the m^{th} element of row r of the column SHIFT in HDU h_{ideal}.

5. The response $\mathcal{R}$ is obtained from w, $\mathcal{R}_r$, $\mathcal{R}_{r+1}$, and s:
   For $j = 1, 2, \ldots, 4096$,

   (a) If

$$s[j-1] < 0.1, \tag{48}$$

   then the shifted pulse heights

$$j_- = j \text{ and} \tag{49}$$
$$j_+ = j, \tag{50}$$

   the fractional amount of the shifted pulse heights

$$\Delta_- = 0 \text{ and} \tag{51}$$
$$\Delta_+ = 0, \tag{52}$$

   and the out-of-bounds flags

$$f_- = 1 \text{ and} \tag{53}$$
$$f_+ = 1. \tag{54}$$

   (b) If

$$s[j-1] \geq 0.1, \tag{55}$$

   then

   i.

$$x_- = j - (1-w)\, s[j-1] \tag{56}$$
$$x_+ = j + w s[j-1] \tag{57}$$
$$j_- = \text{int}\,(x_-) \tag{58}$$
$$j_+ = \text{int}\,(x_+) \tag{59}$$
$$\Delta j_- = x_- - j_- \tag{60}$$
$$\Delta j_- = x_+ - j_+ \tag{61}$$

   ii. If

$$j_- \geq 1 \text{ and} \tag{62}$$
$$j_- \leq 4095, \tag{63}$$

   then

$$f_- = 1. \tag{64}$$

   iii. If

$$j_- < 1 \text{ or} \tag{65}$$
$$j_- > 4095, \tag{66}$$

   then

$$f_- = 0. \tag{67}$$

iv. If

$$j_+ \geq 1 \text{ and} \tag{68}$$
$$j_+ \leq 4095, \tag{69}$$

then

$$f_+ = 1. \tag{70}$$

v. If

$$j_+ < 1 \text{ or} \tag{71}$$
$$j_+ > 4095, \tag{72}$$

then

$$f_+ = 0. \tag{73}$$

(c) The shifted and interpolated response

$$\mathcal{R}[j-1] = \frac{1}{f_- w + f_+(1-w)} \Big[f_- w \Big((1-\Delta j_-)\, \mathcal{R}_r[j_- - 1] + \Delta j_- \mathcal{R}_r[j_-] \Big) + \tag{74}$$
$$f_+ (1-w) \Big((1-\Delta j_+)\, \mathcal{R}_{r+1}[j_+ - 1] + \Delta j_+ \mathcal{R}_{r+1}[j_+] \Big) \Big]. \tag{75}$$

4.2 Scatter matrix

To obtain a position-dependent scatter matrix $S[m,k]$:[6]

1. The scatter matrix data are read:

   (a) Front-illuminated CCD:
   If the `CCD_ID` corresponds to a front-illuminated CCD, then for $m = 0, 1, \ldots, N_{\rm phachan} - 1$,[7]

   i. At and below the peak of the scatter matrix function:
   For $k = -500, -499, \ldots, \text{int}(\texttt{L1\_POS}[m])$,

$$\mathcal{S}[m, k+500] = -|\texttt{L1ALPH1}| \log \left(1 + \left(\frac{k - \texttt{L1\_POS}[m]}{\texttt{L1\_WIDTH}[m]} \right)^2 \right). \tag{76}$$

   ii. Above the peak of the scatter matrix function:
   For $k = \text{int}(\texttt{L1\_POS}) + 1, \text{int}(\texttt{L1\_POS}[m]) + 2, \ldots, 500$,

$$\mathcal{S}[m, k+500] = |\texttt{L1ALPH2}| \log \left(1 + \left(\frac{k - \texttt{L1\_POS}[m]}{\texttt{L1\_WIDTH}[m]} \right)^2 \right). \tag{77}$$

   `L1ALPH1` and `L1ALPH2` are keywords and `L1_POS`, `L1_WIDTH`, and `PHACHAN` are columns in HDU $h_{\rm scatter,fi}$ of the `infile`. The logarithm is used so that the contour interpolation in section 4.2.2 is performed on the log of scattering matrix function. The sign change of $\mathcal{S}$ from one side of the peak (eqn. 76) to the other (eqn. 77) is also important for contour interpolation.

[6]There are $N_{\rm phachan}$ pulse height channels m and $N_{\rm scatter} = 1001$ scatter channels k. The number of scatter channels is hard coded.

[7]For HDU $h_{\rm scatter,fi}$ of the CALDB file acisD2000-01-29p2_respN0006.fits, the $N_{\rm phachan} = 16$ values of `PHACHAN`$[m]$ are 40, 60, 80, 100, 128, 180, 230, 300, 380, 460, 512, 670, 1025, 1535, 2050, and 3100.

(b) Back-illuminated CCD:
If the CCD_ID corresponds to a back-illuminated CCD, then for $m = 0, 1, \ldots, N_{\mathrm{phachan}} - 1$,[8]

i. If

$$\texttt{G1\_FWHM}[m] \quad < \quad 1.5, \tag{78}$$

then

$$\texttt{G1\_AMPL}[m] \quad = \quad \texttt{G1\_AMPL}[m] \left(\frac{\texttt{G1\_FWHM}[m]}{1.5} \right) \text{ and} \tag{79}$$

$$\texttt{G1\_FWHM}[m] \quad = \quad 1.5. \tag{80}$$

ii. At and below the peak of the scatter matrix function:
For $k = -500, -499, \ldots, \mathrm{int}(\texttt{G1\_POS}[m])$,

$$\mathcal{S}[m, k+500] \quad = \quad -\frac{1}{2} \left((k - \texttt{G1\_POS}[m]) \frac{2.35482}{\texttt{G1\_FWHM}[m]} \right)^2. \tag{81}$$

iii. Above the peak of the scatter matrix function:
For $k = \mathrm{int}(\texttt{G1\_POS}) + 1, \mathrm{int}(\texttt{G1\_POS}[m]) + 2, \ldots, 500$,

$$\mathcal{S}[m, k+500] \quad = \quad \frac{1}{2} \left((k - \texttt{G1\_POS}[m]) \frac{2.35482}{\texttt{G1\_FWHM}[m]} \right)^2. \tag{82}$$

G1_AMPL, G1_FWHM, G1_POS, and PHACHAN are columns in HDU $h_{\mathrm{scatter,bi}}$ of the infile.[9] Again, equations 81 and 82 are logarithms of the scatter matrix function and the sign change above the peak is intentional.

2. The scatter matrix is (contour) interpolated to the channel grid:

(a) Front-illuminated CCD:
If the CCD_ID corresponds to a front-illuminated CCD, then for $j = 1, 2, \ldots, 4096$,

i. If

$$j \quad < \quad \texttt{PHACHAN}[0], \tag{83}$$

then

$$S[j-1, k+500] \quad = \quad \mathcal{S}[0, k+500] \tag{84}$$

for $k = -500, -499, \ldots, 500$.

ii. If

$$j \quad \geq \quad \texttt{PHACHAN}[N_{\mathrm{phachan}} - 1], \tag{85}$$

then

$$S[j-1, k+500] \quad = \quad \mathcal{S}[\texttt{PHACHAN}[N_{\mathrm{phachan}} - 1] - 1, k+500] \tag{86}$$

for $k = -500, -499, \ldots, 500$.

[8]For HDU $h_{\mathrm{scatter,bi}}$ of the CALDB file acisD2000-01-29p2_respN0006.fits, the $N_{\mathrm{phachan}} = 20$ values of PHACHAN$[m]$ are 40, 60, 80, 100, 114, 128, 142, 156, 170, 180, 230, 300, 380, 460, 512, 670, 1025, 1535, 2050, and 3100.

[9]The data in the columns G2_AMPL, G2_FWHM, and G2_POS of HDU $h_{\mathrm{scatter,bi}}$ of the infile are not used because the values of G2_AMPL$[m] = 0$ for all m, at least for the CALDB file acisD2000-01-29p2_respN0006.fits.

iii. If there is some m such that

$$j \geq \texttt{PHACHAN}[m] \text{ and} \quad (87)$$

$$j < \texttt{PHACHAN}[m+1], \quad (88)$$

then

$$S[j-1, k+500] = \left(\frac{j - \texttt{PHACHAN}[m]}{\texttt{PHACHAN}[m+1] - \texttt{PHACHAN}[m]}\right) \times \quad (89)$$

$$\left(\mathcal{S}[m+1, k+500] - \mathcal{S}[m, k+500]\right) + \mathcal{S}[m, k+500] \quad (90)$$

for $k = -500, -499, \ldots, 500$.

iv. At and below the peak of the scatter matrix function:
For $k = -500, -499, \ldots, \mathrm{int}(\texttt{L1\_POS}[m])$,

$$S[j-1, k+500] = e^{S[j-1,k+500]}. \quad (91)$$

The use of the exponential function removes the use of the logarithm in equation 76.

v. Above the peak of the scatter matrix function:
For $k = \mathrm{int}(\texttt{L1\_POS}) + 1, \mathrm{int}(\texttt{L1\_POS}[m]) + 2, \ldots, 500$,

$$S[j-1, k+500] = e^{-S[j-1,k+500]}. \quad (92)$$

The sign change in equation 105 relative to equation 104 removes the sign change in equation 77 relative to equation 76.

(b) Back-illuminated CCD:
If the CCD_ID corresponds to a back-illuminated CCD, then for $j = 1, 2, \ldots, 4096$,

i. If

$$j < \texttt{PHACHAN}[0], \quad (93)$$

then

$$S[j-1, k+500] = \log\left(\texttt{G1\_AMPL}[0]\right) + \mathcal{S}[0, k+500] \quad (94)$$

for $k = -500, -499, \ldots, 500$.

ii. If

$$j \geq \texttt{PHACHAN}[N_{\mathrm{phachan}} - 1], \quad (95)$$

then

$$S[j-1, k+500] = \log\left(\texttt{G1\_AMPL}[N_{\mathrm{phachan}} - 1]\right) + \quad (96)$$

$$\mathcal{S}[\texttt{PHACHAN}[N_{\mathrm{phachan}} - 1] - 1, k+500] \quad (97)$$

for $k = -500, -499, \ldots, 500$.

iii. If there is some m such that

$$j \geq \texttt{PHACHAN}[m] \text{ and} \quad (98)$$

$$j < \texttt{PHACHAN}[m+1], \quad (99)$$

then

$$S[j-1,k+500] = \left(\frac{j-\texttt{PHACHAN}[m]}{\texttt{PHACHAN}[m+1]-\texttt{PHACHAN}[m]}\right)\times \quad (100)$$

$$\left[\left(\texttt{G1\_AMPL}[m+1,k+500]-\texttt{G1\_AMPL}[m,k+500]\right)+ \quad (101)$$

$$\left(\mathcal{S}[m+1,k+500]-\mathcal{S}[m,k+500]\right)\right]+ \quad (102)$$

$$\texttt{G1\_AMPL}[m,k+500]+\mathcal{S}[m,k+500] \quad (103)$$

for $k = -500, -499, \ldots, 500$.

iv. At and below the peak of the scatter matrix function:
For $k = -500, -499, \ldots, \mathrm{int}(\texttt{G1\_POS}[m])$,

$$S[j-1,k+500] = e^{S[j-1,k+500]}. \quad (104)$$

v. Above the peak of the scatter matrix function:
For $k = \mathrm{int}(\texttt{G1\_POS}[m])+1, \mathrm{int}(\texttt{G1\_POS}[m])+2, \ldots, 500$,

$$S[j-1,k+500] = e^{-S[j-1,k+500]}. \quad (105)$$

3. The scatter matrix is normalized:
For $j = 1, 2, \ldots, 4096$:

(a)

$$S_{\mathrm{tot}} = \sum_{k=-500}^{500} S[j-1,k+500]. \quad (106)$$

(b) For $k = -500, -499, \ldots, 500$,

$$S[j-1,k+500] = \frac{S[j-1,k+500]}{S_{\mathrm{tot}}}. \quad (107)$$

4.3 Convolve ideal RMF and scatter matrix

To obtain the RMF:

1. The ideal RMF and scatter matrix are convolved:
For $j = 1, 2, \ldots, 4096$,

(a) If

$$j < 501, \quad (108)$$

then

$$k_{\mathrm{lo}} = 1-j. \quad (109)$$

(b) If

$$j \geq 501, \quad (110)$$

then

$$k_{\mathrm{lo}} = -500. \quad (111)$$

(c) If

$$j < 3596, \tag{112}$$

then

$$k_{\rm hi} = 500. \tag{113}$$

(d) If

$$j \geq 3596, \tag{114}$$

then

$$k_{\rm hi} = 4096 - j. \tag{115}$$

(e)

$$R[i, j-1] = \sum_{k=k_{\rm lo}}^{k_{\rm hi}} \mathcal{R}[(j-1)+k]S[j-1, k+500]. \tag{116}$$

5 TBD

1. PI RMFs
2. Nonstandard `channel` ranges or binning
3. Weighting for multiple regions