HRC-S/LETG Filter Vignetting Function

(ASC Technical Reference)

http://space.mit.edu/ASC/docs/LETGS/hrcs_vign.ps.gz

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1 Description of the Problem

The central region of the High Resolution Camera spectroscopic array (HRC-S) has filter boundaries on the left and right of center (regions 0 and 1 in Figure 1). The filter is offset from the detector Micro-Channel Plate (MCP) by 12 mm toward the mirror (HRMA). The effective area at any particular energy will change gradually near the boundary, since the beam is still converging where the cone of rays crosses the filter — part of the beam will pass through one type of filter, and the rest through the other, before converging at the focus (see Figure 2). Even a physically sharp filter boundary is blurred into a shadow at the focus. Such shadowing of an image by an aperture is commonly referred to as vignetting.

For any point, y, near the 0-1 boundary shadow in the focal plane, the effective area as a function of wavelength, $A(\lambda, m)$, can be written as a weighted sum of the areas for rays passing through each part of the filter:

$$A(\lambda, y, m) = \sum_i M_i(\lambda)\, G_i(\lambda, m)\ \{f_i(y)Q_1(\lambda) + [1 - f_i(y)]Q_0(\lambda)\}. \qquad (1)$$

Here,

M_i is the mirror area for shell i (traditionally indexed with 1, 3, 4, and 6 from the outer to inner HRMA pairs);

$G_i(\lambda, m)$ is the grating efficiency as a function of diffraction order, m, and wavelength.,

$Q_k(\lambda)$ is the product of the filter transmission and detector quantum efficiency for filter k ($k = 0$ or 1; see the figure); and

$f_i(y)$ is the fraction of the cone for shell i which passes through filter region 1.

```
+-----------------+ +-------------------+ +-------------------+
|      2          | |         0         | |      2            |
+-----------------+ +----+       +------+ +-------------------+
|                 | |    |       |      | |                   |
|                 | |    |  ox   |      | |                   |
|                 | |    |       |      | |                   |
|     3           | | 1  |       |  1   | |       3           |
|                 | |    |       |      | |                   |
+-----------------+ +-----+--------+----+ +-------------------+
                              L<--|-->R
```

Figure 1: This is a schematic of the HRC-S MCP and filter geometry and is based on a memo dated March 14 1996 from Mike Juda with title: "New HRC UV/Ion Shields". The geometric center of the middle facet is marked by an "x", and the optical axis by an "o". Regions labeled $0-3$ represent different filter transmissivities. The 0-1 boundaries on the central region are symmetric about the optical axis. The filter lies about 10 mm along the positive X axis from the center MCP. It lies in a flat plane. The positive AXAF y-axis is to the right, positive z is up, and positive x is out of the page.

In order to provide an accurate effective area function through the filter shadow for LETGS spectra at energies and diffraction orders which project near the filter boundary, the functions, $f_i(y)$, must be specified.

2 Necessary Parameters

The relevant calibration parameters are shown in Figure 3. They are:

X_B Mirror hyperboloid shells X-coordinate at the back (exit).

X_F Mirror hyperboloid shells X-coordinate at the front (entrance).

$R_{\mathrm{B}i}$ Mirror hyperboloid shell radius at the back (exit) for shell i.

$R_{\mathrm{F}i}$ Mirror hyperboloid shell radius at the front (entrance) for shell i.

X_f The filter offset from the detector micro-channel plate. Positive X is towards the mirror.

Y_R The distance from the plate center to the filter boundary on the $+y$ side (right).

Y_L The distance from the plate center to the filter boundary on the $-y$ side (Left).

There are two primary observational parameters relevant to determining f_i:

p This is the "impact parameter": it is defined as the distance in y from the filter boundary to the center of the converging cone (see Figure 3).

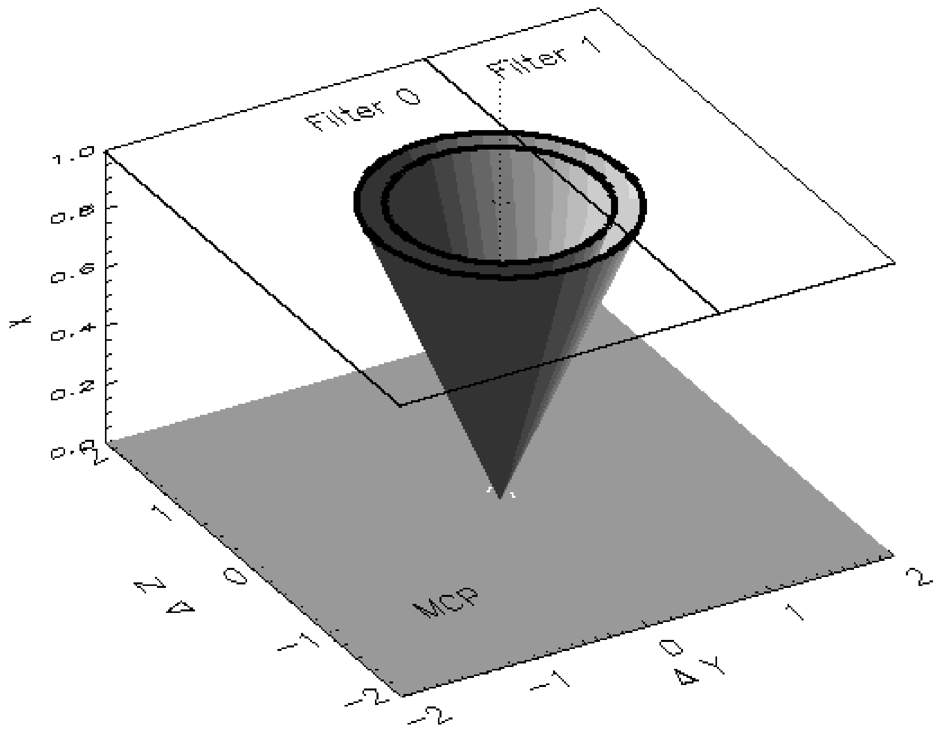


Figure 2: This is a schematic of the converging cone of rays where it passes through the filter and is imaged onto the MCP. The signal on the MCP is the result of combining some fraction of the beam which passes through "Filter 1" with the remaining fraction from "Filter 0".

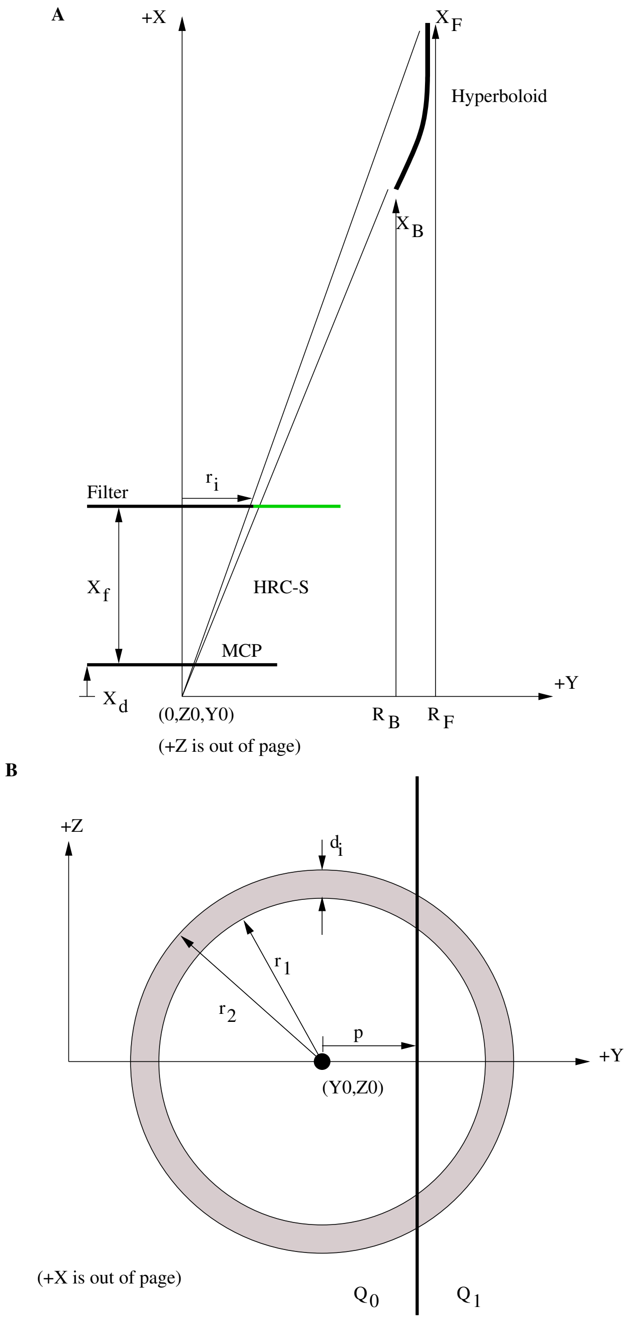


Figure 3: This is a schematic of the system showing a cross-section of the beam for one HRMA shell. Views are shown in both the $y-x$ plane (panel A) and the $y-z$ plane at the filter (panel B).

X_d This is the detector defocus along the x axis. It is nominally small (but not zero). This parameter is also shown in Figure 3).

The impact parameter is defined in the plane of the filter by the distance, y, from the cone center to the filter boundary. We will transform from y to p in the definition of f.

It must be realized that the wavelength is also a function of y through diffraction — $\lambda = \lambda(y, m)$; this coordinate transformation (and its other dimensional dependencies) will not be discussed here. The appropriate wavelength in Equation 1 is for the value of y at the cone center.

An additional detail is that the MCP quantum efficiency (QE) depends upon the ray's angle of incidence. The MCP pores are tilted by about 6° perpendicularly to the dispersion (tilt is in the direction parallel to AXAF z-axis). The convergence angle of the outer HRMA shell is about 3°. Over the range of -3 to $+3$ degrees, the beam thus makes angles of 9 to 3 degrees relative to the pore angle. The MCP efficiency (at 1.49 keV) ranges from about 1.15 to 0.85 in this interval (See Fig 6.8, p.132, "AXAF Observatory Guide, Rev 1.0, Oct 1997"). Hence, the affect should not be ignored. It will be approximated with a linear function, since the forementioned figure shows a reasonably linear relationship. The relative quantum efficiency $(Q(\theta, E)/Q(0, E))$ will be assumed to be of the form $q = q_0 + q_1\theta$, in which the q_i are calibration parameters and θ is the angle between the beam and the pore axis. For the current geometry, this angle is determined primarily by the component of the ray in the z-direction, parallel to the pore-tilt direction. There is only a very weak dependence on y, and it will be ignored.

Values adopted for the parameters are given in Table 1. Using these values, the largest grating ring will project to a radius of 0.800 mm at the HRC-S filter, and have an annular width of 0.021 mm. Aberrations which broaden this width will be ignored since they do not affect the fractional area, f_i. At the positions of the filter boundaries, the LETG first order wavelengths are about 18 Å for an on-axis source (0.70 keV). Typical offsets (about 4 mm, or 1 arcmin) can alter this by about ±4 Å (±0.16 keV), which is enough to cause significant change in sensitivity across the filter.

3 Detailed Solution

The functions to be determined, $f_i(p)$, are the fractional area of the converging cone passing through filter region 1 (as defined by Figure 1). The shape will be treated as an ideal annulus which is the projection of the grating ring onto the filter. Detailed affects, such as images of individual facets and obscuration by support structures, will be ignored. Hence we need to specify the area of the segment of an annulus, but weighted by the QE angular dependence. This area will be denoted as

$$A_a = A_a(r_1, r_2, p) \tag{2}$$

Table 1: Adopted Parameter Values

Parameter	Value
$R_{\mathrm{B}i}$	560.86506, 451.45717, 398.51324, 296.13584 mm
$R_{\mathrm{F}i}$	598.52343, 481.77216, 425.27403, 316.02294 mm
X_{B}	9179.8 mm
X_{F}	10018.0 mm
X_f	12.33 mm
X_d	0.1 mm
Y_R	15.7 mm
Y_L	-15.7 mm
q_0	1.00
q_1	-2.865 rad^{-1}

in which r_1 is the inner radius of the annulus, r_2 is the outer radius (both as projected from the grating onto the filter), and p is the impact parameter — the distance from the center of the annulus on the filter to the line truncating the segment (the filter boundary). By symmetry, we have

$$A_a(p < 0) = 2A_a(0) - A_a(-p) \tag{3}$$

(in which the radii parameters have been suppressed). The annular area can in turn be expressed as the difference between the areas of two circular segments, one for the outer radius, and one for the inner. The circular segment area will be denoted as

$$A_c = A_c(r, p) \tag{4}$$

in which r is the radius of the circle, and p is again an impact parameter of the truncating line. Again by symmetry,

$$A_c(p < 0) = 2A_c(0) - A_c(-p). \tag{5}$$

The area of the annulus becomes (being explicit about the form on the domain of the impact parameter):

$$\begin{aligned} A_a(r_1, r_2, p) &= A_c(r_2, p), & r_1 \le p < r_2 \\ &= A_c(r_2, p) - A_c(r_1, p), & 0 \le p < r_1 \\ &= 2\left[A_c(r_2, 0) - A_c(r_1, 0)\right] - \left[A_c(r_2, -p) - A_c(r_1, -p)\right], & p < 0 \end{aligned} \tag{6}$$

To specify A_c, we must integrate the area element over the region of a segment, weighted by the QE angular dependence:

$$A_c(r, p) = \int_p^r dy \int_{-z(y)}^{z(y)} dz\, q(z) \tag{7}$$

$$z(y) = (r^2 - y^2)^{\frac{1}{2}} \tag{8}$$

The area weighting function is the MCP QE angular dependence expressed as a function of z instead of θ. The component of the angle parallel to the pore tilt is found by noticing that $\tan\theta = z/X_f$. Since we are dealing with small angles, we have $\theta \approx z/X_f$, so we still have a linear dependence of the QE on z:

$$q = q_0 + \frac{q_1}{X_f} z. \tag{9}$$

Since the angular dependence is linear and symmetric about the center of the annulus, anything gained above in the integral is lost below and there is no depencence upon q_1 in the result. Also, since the dependence was normalized, q_0 is necessarily 1. (Even if it were not, its dependence would cancel when forming a fractional area.)

The integral is analytic and becomes

$$A_c(r,p) = q_0\, r^2 \left[\frac{\pi}{2} - \tilde{p}(1-\tilde{p}^2)^{1/2} - \arcsin(\tilde{p})\right], \tag{10}$$

in which $\tilde{p} = p/r$.

The desired functions, $f(p)$, are then

$$f(p) = \frac{A_a(r_1, r_2, p)}{q_0 \pi (r_2^2 - r_1^2)}. \tag{11}$$

The denominator is the area of an annulus (weighted by the pore-tilt affect, but normally, $q_0 = 1$). Note that each HRMA (or grating) shell will have its own unique values of r_1, r_2, and p. The $f_i(p)$ functions could be pre-computed for nominal flight parameters; the only observationally dependent parameter is the defocus, and this should be relatively stable.

4 Simplifications

For energies above about 1 keV, the differences in filter transmissions are small. Hence we only need to be concerned with low-energies (or low diffraction orders). At these energies, the relative contributions from each mirror shell do not change. That is,

$$\mathcal{M}(\lambda) = M_i(\lambda)/\mathcal{A}_i, \tag{12}$$

in which $\mathcal{A}_i$ is a shell's geometric area, and $\mathcal{M}$ is independent of the shell, i. We will also assume that the grating efficiencies for each shell are equal. In this case, Equation 1 becomes

$$A(\lambda, p, m) = \mathcal{M}(\lambda) G(\lambda, m) Q_1(\lambda) \left\{ \sum_i \mathcal{A}_i f_i(p) + \mathcal{Q}(\lambda) \sum_i \mathcal{A}_i [1 - f_i(p)] \right\}. \tag{13}$$

We have additionally defined $\mathcal{Q}(\lambda) = Q_0/Q_1$.

An advantage of this form is that the summation terms could be precalculated. An additional assumption which could be safely made is that over the small wavelength region of interest encompassing the filter shadow, the efficiencies the mirror and grating are constant. The only terms which are a function of wavelength are the filter transmissions, and if we are not spanning an absorption edge, these will be effectively constant. In most cases, the only terms which depend upon position are geometric and can be pre-calculated.

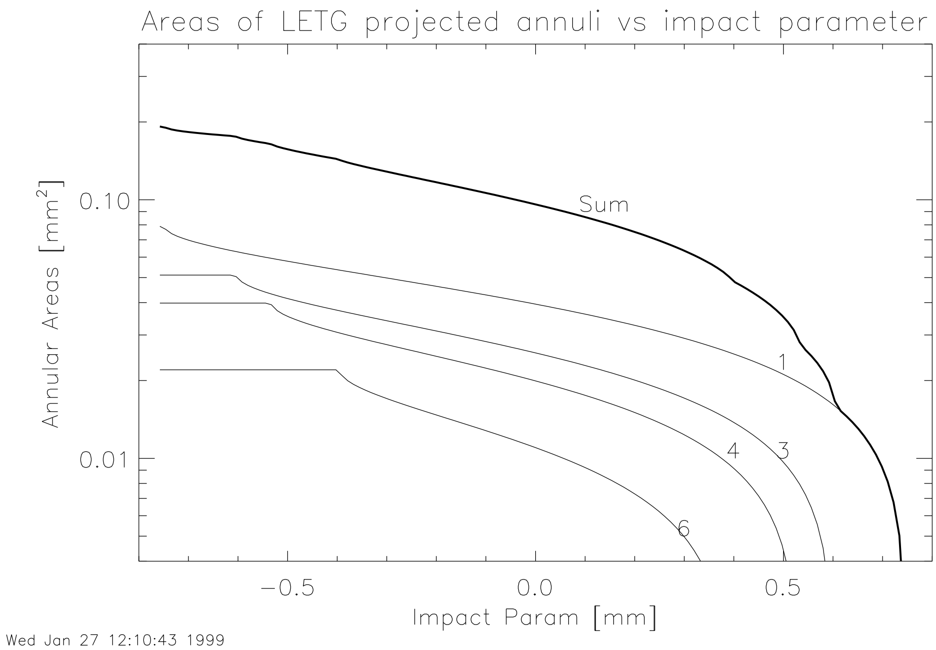
5 Unresolved Issues

- New ARD are implied and need formal specification (Analysis Reference Data)
- The outer boundary, where the inner and outer MCP's meet, there is also a filter boundary and shadow. Needs analysis. Some issues:
  - The MCP gap is 1.47 mm across. The beam diameter on the filter is about 1.6 mm across. Hence, there is a smaller vignetting region. What is fractional area for impact parameters where 2 regions are seen by MCP? Is small range.
  - Outer MCP's tilted; what is affect on pore-tilt dependence. Guess is that predominant affect is still w/ z-component of beam, and not w/ y. Normalization may change, since mean component in y is different. MCP tilt makes outer segment depencence like inner segment, so any affect is more important on inner segment where beam is at greater angle relative to MCP normal.
  - LESF boundary (0-2) has biggest affect, since filters differ the most.

6 Example

Figure 4 shows an example calculation of the $f_i(p)$ functions for each shell and their sum for an impact parameter range which passes the entire ensemble image across the filter.

The IDL programs used to compute the figure follows.



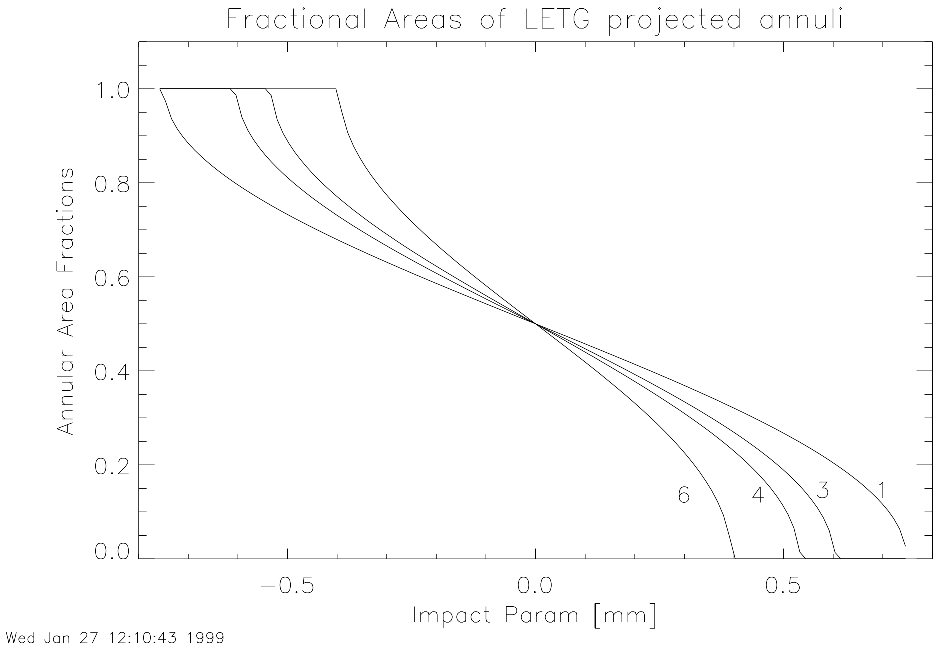


Figure 4: The areas and relative area fractions defined by are shown against the impact parameter for the four HRMA shells as imaged onto the HRC-S filter.

```
PRO hrcs_filt, areas

                        ;;;;;;;; Data:
   X_d = 0.1            ; defocus; MCP is nominally ahead of Rowland by 0.1mm
   X_f = 12.3           ; filter offset from MCP

; ---- HRMA Hyperboloid radii and positions of front and back.
   hs_HAf = 10018.0d0           ; Hyperbola Axial distance to focus, front
   hs_HAb = 9179.8d0            ; Hyperbola Axial distance to focus, back
                                ; Hyperbola Radius, front
   hs_HRf = [598.52343d0, 481.77216, 425.27403, 316.02294]
                                ; Hyperbola Radius, back
   hs_HRb = [560.86506d0,  451.45717, 398.51324, 296.13584]
; ---------------------------------------------------------------

                                ; compute r_i,  the projected inner
                                ;  radii OF the annuli

   r1_i = hs_HRf / hs_HAf * (X_f + X_d) ; inner radii
   r2_i = hs_HRb / hs_HAb * (X_f + X_d) ; outer radii

                                ; Impact parameter range is thus
                                ; -max(r2) to +max(r2)
   p_min = -max(r2_i)
   p_max = -p_min

   npts = 128
   p_param = findgen(npts)/npts * (p_max-p_min) + p_min

   A1 = annular_segment_area(r1_i[0], r2_i[0], p_param)
   A3 = annular_segment_area(r1_i[1], r2_i[1], p_param)
   A4 = annular_segment_area(r1_i[2], r2_i[2], p_param)
   A6 = annular_segment_area(r1_i[3], r2_i[3], p_param)

   Annulus_area = !pi * ( r2_i^2 - r1_i^2 )
   f1 = A1 / Annulus_area[0]
   f3 = A3 / Annulus_area[1]
   f4 = A4 / Annulus_area[2]
   f6 = A6 / Annulus_area[3]

   areas = {p_param:p_param, $
            A1:A1, A3:A3, A4:A4, A6:A6, $
            f1:f1, f3:f3, f4:f4, f6:f6}
END
```

```
FUNCTION circular_segment_area, radius, impact_param
;+
; NAME:
;
; circular_segment_area
;
; PURPOSE:
;
;  Caluclate the area of a segment of a circle, given the impact parameter.
;
; CATEGORY:
;
; CALLING SEQUENCE:
;
; result = circular_segment_area(radius, impact_parameter)
;
; INPUTS:
;
; radius: (real) radius of circle
;
; impact_parameter (real array) distance of segment from center of
; circle.
;
; OPTIONAL INPUTS:
;
; KEYWORD PARAMETERS:
;
; OUTPUTS:
;
;  result (real array) = area of segment (region between circle and line
;  on side of line furthest from center of circle)
;
; OPTIONAL OUTPUTS:
;
; none
;
; COMMON BLOCKS:
;
; none
;
; SIDE EFFECTS:
;
; none
;
; RESTRICTIONS:
;
```

```
; radius > 0
;
; PROCEDURE:
;
; simple geometry
;
; EXAMPLE:
;
; MODIFICATION HISTORY:
;
; 981219 dph
;-

   np = n_params()

   IF np NE 2 THEN BEGIN
      print, 'Usage: result=circular_segment_area(radius, impact_param)'
      return, -1
   ENDIF

   IF radius LT 0 THEN BEGIN
      print, 'radius must be >=0'
      return, -1
   ENDIF

   n_el = n_elements(impact_param)
   result = fltarr(n_el)

   list_outside = where( impact_param GE  radius)
   IF (size(list_outside))[0] NE 0 THEN BEGIN

      result[list_outside] = 0.0

   ENDIF


   list_pos = where( (impact_param LT radius) $
                     AND $
                     impact_param GE 0.0)

   IF (size(list_pos))[0] NE 0 THEN BEGIN

      d = impact_param[list_pos] / radius

      result[list_pos] = $
       radius^2 * (!pi/2. - d * sqrt(1.0-d^2) - asin(d) )
```

```
   ENDIF

   list_neg = where( (impact_param LT 0.0) $
                     AND (impact_param GT -radius) )

   IF (size(list_neg))[0] NE 0 THEN BEGIN ; use symmetry
                                ; if d<0,
                                ; A_c(-d) = 2*A_c(d=0) - A_c(-d)

      d = -impact_param[list_neg] / radius

      result[list_neg] = radius^2 * (!pi/2 + d * sqrt(1.0-d^2) + asin(d) )


   ENDIF


   list_negout = where( impact_param LE -radius)

   IF (size(list_negout))[0] NE 0 THEN BEGIN

      result[list_negout] =  !pi*radius^2

   ENDIF

   return, result

END
```

```
FUNCTION annular_segment_area, radius_inner, radius_outer, impact_param
;+
; NAME:
; annular_segment_area
;
; PURPOSE:
;
; Compute the area of an annular segment given the inner radius,
; outer radius, and impact parameter.
;
; CATEGORY:
;
; CALLING SEQUENCE:
;
;  area = annular_segment_area(radius_inner, radius_outer, impact_param)
;
; INPUTS:
;
; radius_inner (real): radius of inner annulus
;
; radius_outer (real): radius of outer annulus
;
; impact_param (real array): impact parameter - the
;                            coordinate of the line drawn across the
;                            circle.  This is the distance of the line
;                            from the center of the circle.
;
; OPTIONAL INPUTS:
;
; none
;
; KEYWORD PARAMETERS:
;
; none
;
; OUTPUTS:
;
;  area (real array) : area of annular segment vs impact parameter.
;
; OPTIONAL OUTPUTS:
;
;  none
;
; COMMON BLOCKS:
;
;  none
```

```
;
; SIDE EFFECTS:
;
; none
;
; RESTRICTIONS:
;
;  radius_inner < radius_outer
;
; PROCEDURE:
;
;  Simple geometry. The area of an annular segment is the difference of
;  the area of two circular segments.  Three regimes are defined,
;  depending on the value of the impact_parameter relative to 0,
;  radius_inner, and radius_outer.
;
; EXAMPLE:
;
;
; MODIFICATION HISTORY:
;
; 981219 dph.
;-

   np = n_params()

   IF np NE 3 THEN BEGIN
      print, 'Usage: area = annular_segment_area(radius_inner, radius_outer, impact_param)'
      return, -1
   ENDIF

   IF radius_Inner GT radius_outer THEN BEGIN
      message, /inf, 'Inner radius must be < outer radius'
      return, -1
   ENDIF

   impact_param = [impact_param] ; force to array
   n_el = n_elements(impact_param)
   result = fltarr(n_el)

   list_outside = where(impact_param GE radius_outer)

   IF  (size(list_outside))[0] NE 0 THEN  result[list_outside] = 0.0


   list_between =  where ( (impact_param GE radius_inner) $
```

```
                        AND $
                        (impact_param LT radius_outer) )

IF  (size(list_between))[0] NE 0 THEN BEGIN

   result[list_between] = $
    circular_segment_area(radius_outer, impact_param[list_between])

ENDIF


list_inner = where( (impact_param LT radius_inner) $
                  AND (impact_param GE 0))

IF (size(list_inner))[0] NE 0 THEN BEGIN

   result[list_inner] = $
    circular_segment_area(radius_outer, impact_param[list_inner]) $
    - circular_segment_area(radius_inner, impact_param[list_inner])

ENDIF


list_negative = where( (impact_param LT 0) $
                       AND $
                       (impact_param GE -radius_inner) )

IF (size(list_negative))[0] NE 0 THEN BEGIN

                             ; use symmetry in impact_param
                             ; relative to center.

   result[list_negative] =  $
    (2 * ( $
         circular_segment_area(radius_outer, 0) $
         - circular_segment_area(radius_inner, 0) $
         ))[0] $             ; hack - force single-element vector to scalar
    - ( $
       circular_segment_area(radius_outer, -impact_param[list_negative]) $
       - circular_segment_area(radius_inner, -impact_param[list_negative]) $
      )

ENDIF

list_negbet = where( (impact_param LT -radius_inner) $
                     AND (impact_param GE -radius_outer) )
```

```
  IF (size(list_negbet))[0] NE 0 THEN BEGIN

                              ; use symmetry in impact_param
                              ; relative to center.

     result[list_negbet] =  $
      (2 * ( $
           circular_segment_area(radius_outer, 0) $
           - circular_segment_area(radius_inner, 0) $
           ))[0] $             ; hack - force single-element vector to scalar
      - circular_segment_area(radius_outer, -impact_param[list_negbet])

  ENDIF


  list_negout = where( impact_param LT -radius_outer )

  IF (size(list_negout))[0] NE 0 THEN BEGIN

     result[list_negout] =  $
      2 * ( circular_segment_area(radius_outer, 0) $
            - circular_segment_area(radius_inner, 0) )

  ENDIF


  return, result

END
```