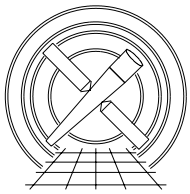

**MIT Kavli Institute** **Chandra X-Ray Center**

**MEMORANDUM**
June 1, 2005

| | |
|---|---|
| **To:** | Jonathan McDowell, SDS Group Leader |
| **From:** | Glenn E. Allen, SDS ACIS Scientist |
| **Subject:** | ACIS Instrument Geometry |
| **Revision:** | 1.0 |
| **URL:** | http://space.mit.edu/CXC/docs/docs.html#instgeom |
| **File:** | /nfs/cxc/h2/gea/sds/docs/notes/check_instgeom/memo_instgeom_1.0.tex |

# 1 Introduction

I tested the accuracy of the computation of the ACIS sky coordinates by writing some independent coordinate transformation functions. One of these functions computes the coordinates DETX and DETY using, in part, the coordinates CHIPX and CHIPY and the information in HDU 3 (of 0–7) of an instrument geometry file (e.g. telD1999-07-23geomN0005.fits). This HDU contains a table of the $x$, $y$ and $z$ LSI coordinates for each corner of each CCD.

The coordinate transformations I use are based on the assumption that an ACIS CCD is a square. As described in this memo, the LSI coordinates of the ACIS CCDs in the CALDB instrument geometry files do not define two-dimensional square detectors, but the differences between the LSI coordinates and ideal squares are small. The inaccuracies in the CALDB data should not affect the computation of the detector (and, hence, sky) coordinates by more than about 0.04 pixel.

# 2 Tests

I compared the coordinates for the lower left-hand (LL), upper left-hand (UL), upper right-hand (UR) and lower right-hand (LR) corners of each CCD with one another to determine whether

1. The four corner points $P_1, \ldots, P_4$ are coplanar,
2. The lengths of each of the four sides $(l_1, \ldots, l_4)$ are identical to the calibrated length, and
3. The angles between each pair of adjacent sides $(\theta_1, \ldots, \theta_4)$ are 90°.

Figure 1 shows a diagram of an ACIS CCD on which the points $P_i$, lengths $l_i$ and angles $\theta_i$ are labeled. The CHIPX and CHIPY coordinates associated with each vertex are also depicted. Since these coordinates apply to the outer edges of the CCD, the coordinates of the center of each pixel are whole numbers in the range from 1 to 1024.

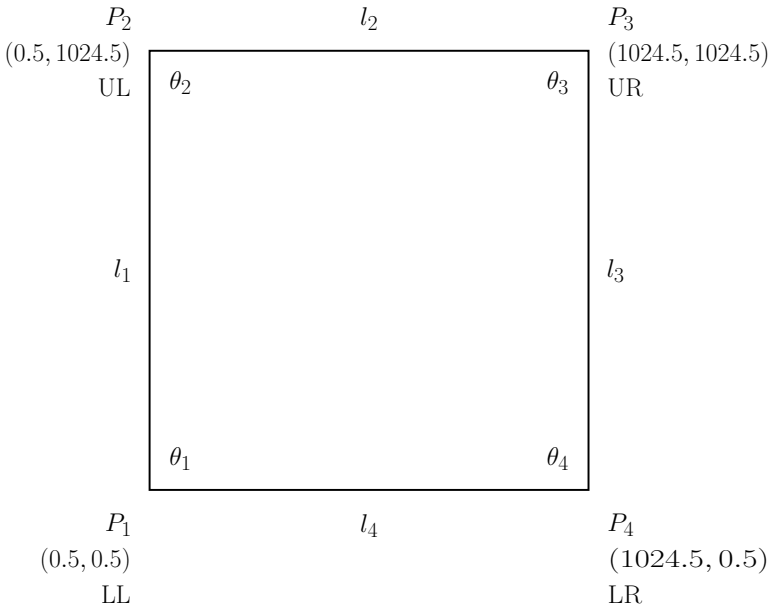


Figure 1: A diagram of an ACIS CCD. The coordinates (CHIPX,CHIPY) and the designation "LL" (lower left-hand), "UL" (upper left-hand), "UR" (upper right-hand) or "LR" (lower right-hand) are indicated for the four vertices $P_1, \ldots, P_4$. The lengths of sides 1, ..., 4 are labeled $l_1, \ldots, l_4$, respectively. The angles between adjacent sides are labeled $\theta_1, \ldots, \theta_4$.

# 3 Results

## 3.1 Coplanarity

The equation of plane through the point $\mathbf{x_0}$ with a unit normal vector $\mathbf{n}$ is

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{x_0}) = 0. \tag{1}$$

To determine if the four corners of a CCD are coplanar:

1. Three of the four corners are used to define a plane,
2. The unit normal vector $\mathbf{n}$ for this plane is computed using the coordinates of these three points, and
3. Equation 1 is used to compute the distance between the coordinates of the fourth corner and the plane.

These three steps were performed for each one of the four possible combinations of three corners. The results of the test are shown in Figure 2. No corner differs from coplanarity by more than about 1.0 $\mu$m (0.04 pixel).

## 3.2 Side lengths

The results for the computation of the lengths $l$ of each side of each CCD are shown in Figure 3. The plots depict the difference between the computed and calibrated lengths. The calibrated lengths (i.e. the values of the keywords XSCALE and YSCALE in HDU 3 of the CALDB file) are included as part of the y-axis label. No CCD has a side whose length differs from the calibrated length by more than about 1.7 $\mu$m (0.07 pixel).

## 3.3 Angles

The results for the computation of the angles $\theta$ between each pair of adjacent sides are shown in Figure 4. No CCD has a corner whose angle differs from a right angle by more than about $0.0023^\circ$ ($4 \times 10^{-5}$ rad).

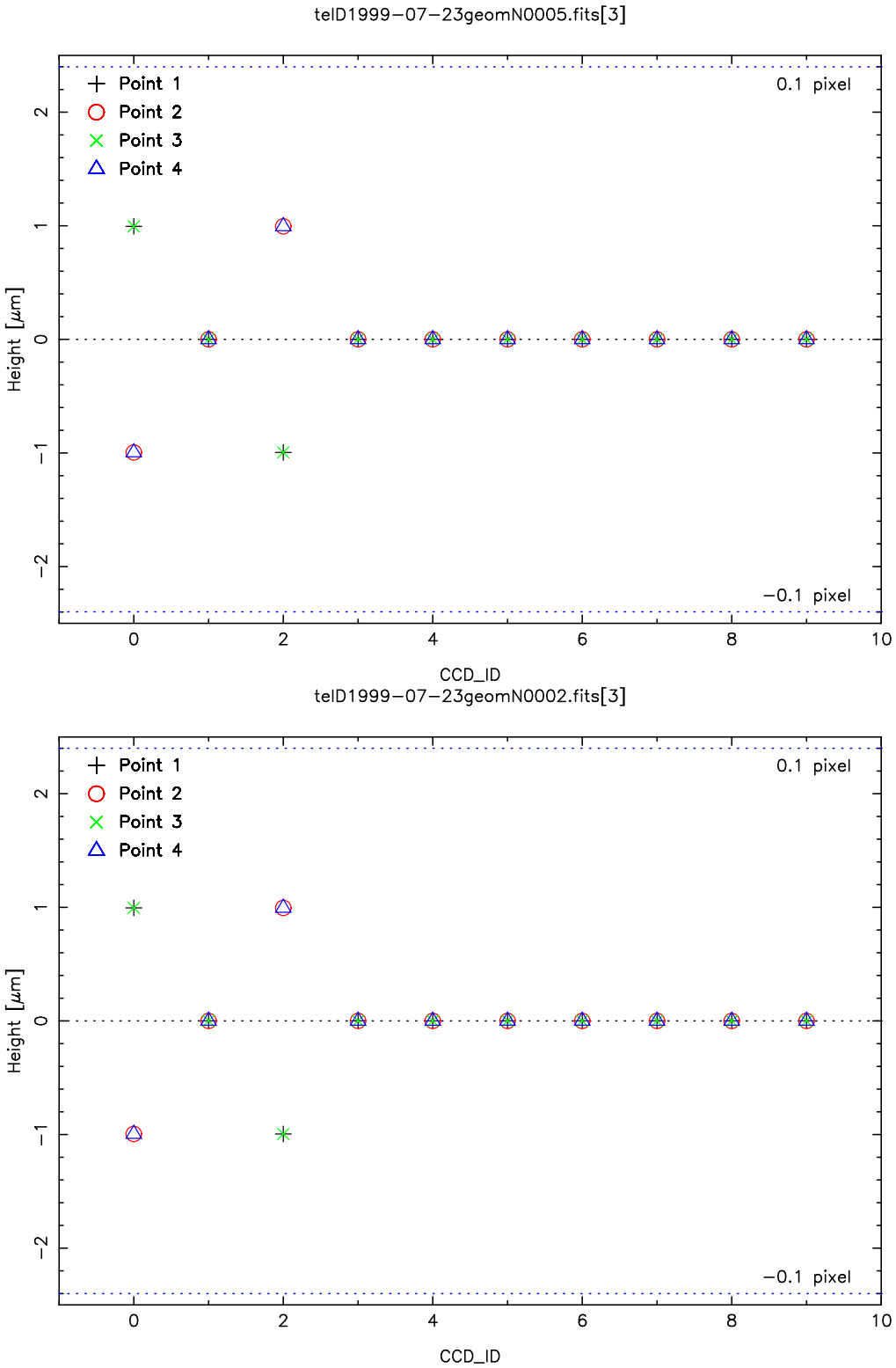


Figure 2: The distance of each corner ($P_1, \ldots, P_4$) of a CCD from a plane defined by the other three corners of the CCD. The upper plot is for the LSI coordinates in the CALDB file telD1999-07-23geomN0005.fits. The lower plot is for telD1999-07-23geomN0002.fits. The dotted lines indicate a difference of 0.1 pixel. The results are the same for these two files: no corner differs from coplanarity by more than about 1.0 $\mu$m (0.04 pixel).

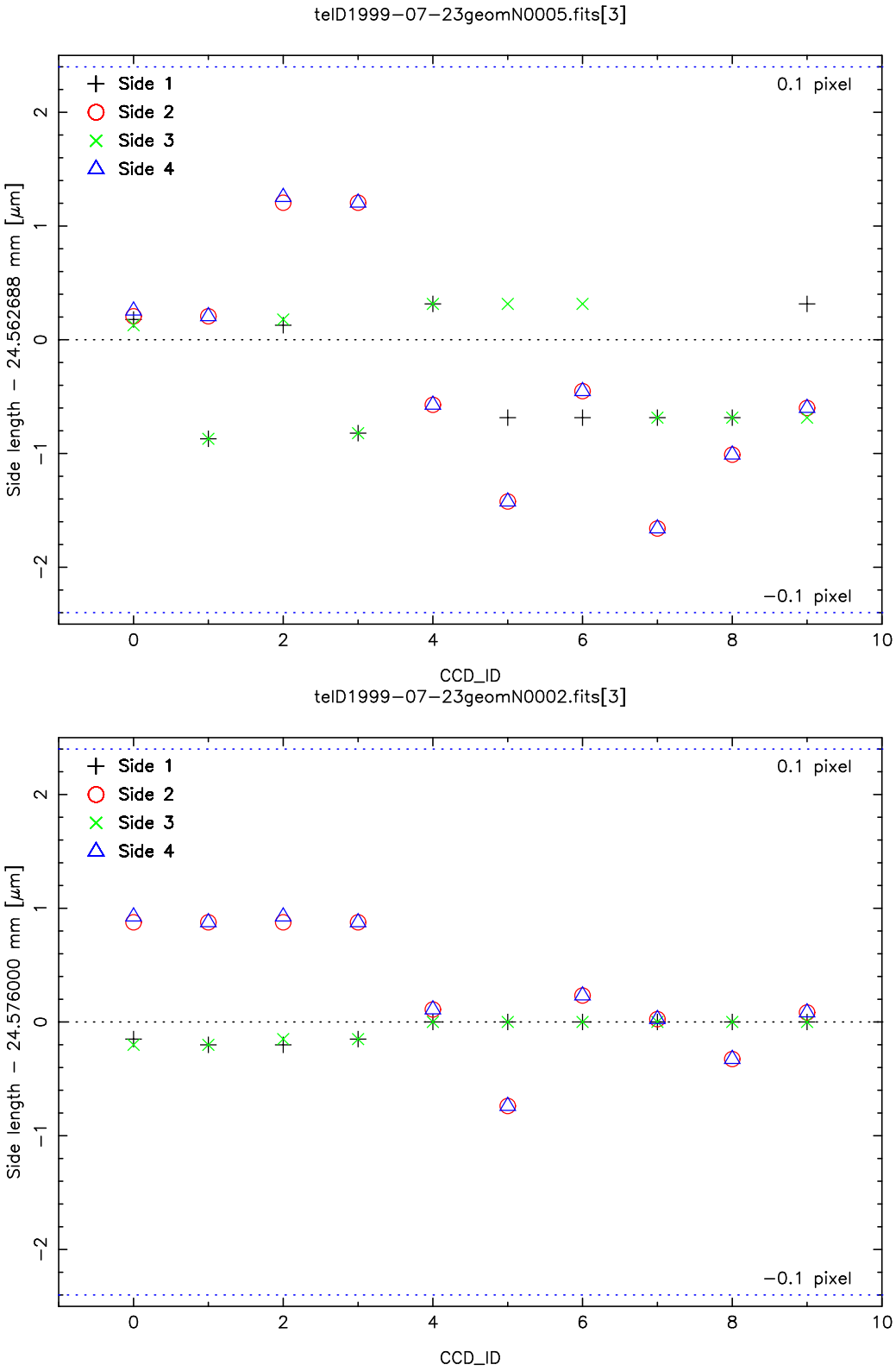


Figure 3: The difference between the length of each side of a CCD ($l_1$, ..., $l_4$) and the calibrated length. The upper plot is for the LSI coordinates in the CALDB file telD1999-07-23geomN0005.fits. The lower plot is for telD1999-07-23geomN0002.fits. The dotted lines indicate a difference of 0.1 pixel. No CCD has a side whose length differs from the calibrated length by more than about 1.7 $\mu$m (0.07 pixel).

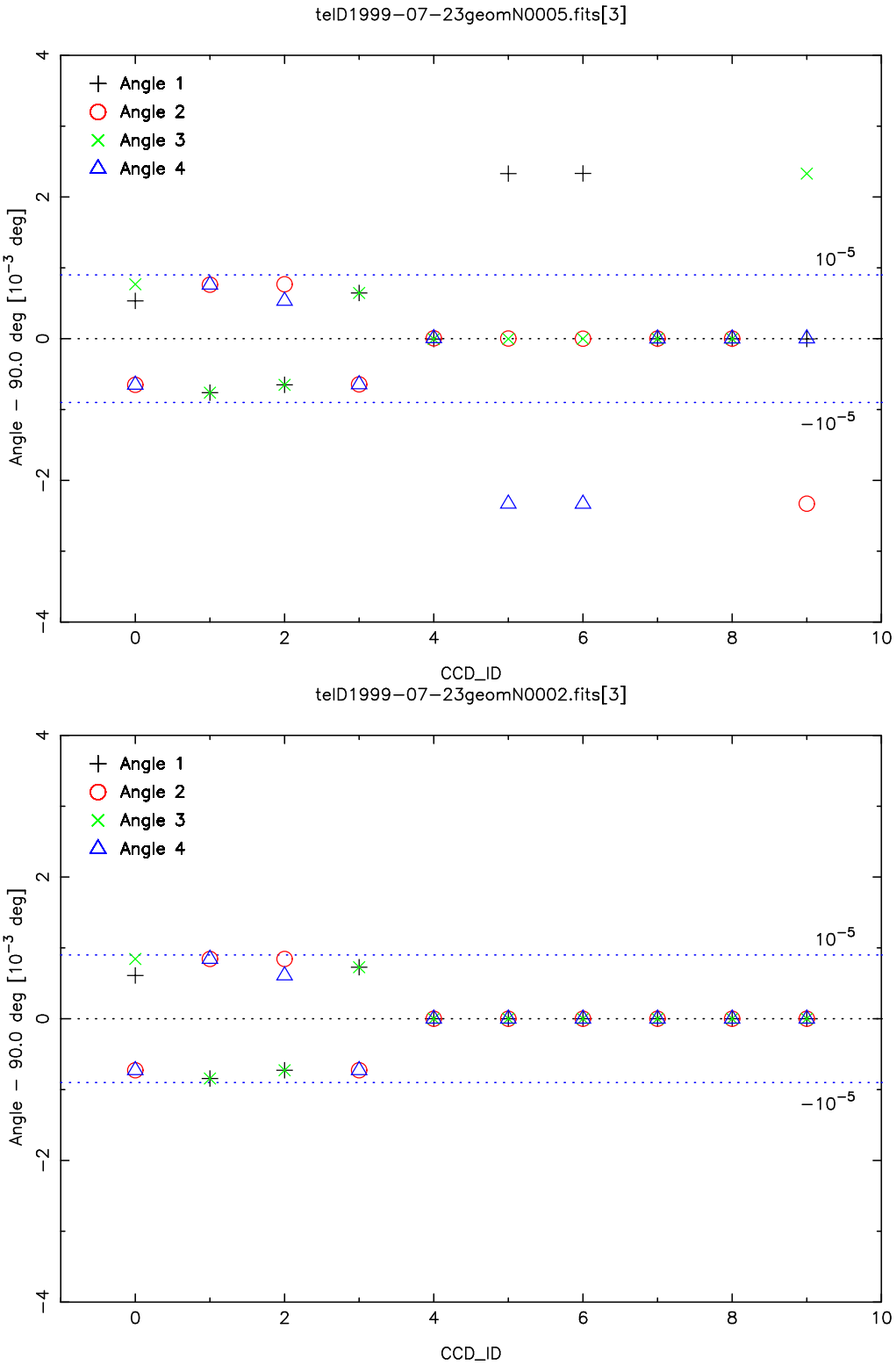


Figure 4: The differences between the computed values for the angles $\theta_1, \ldots, \theta_4$ and the expected value of 90°. The upper plot is for the LSI coordinates in the CALDB file telD1999-07-23geomN0005.fits. The lower plot is for telD1999-07-23geomN0002.fits. The dotted lines indicate a fractional difference of $10^{-5}$ (i.e. $9 \times 10^{-4}$ deg). No CCD has a corner whose angle differs from a right angle by more than about 0.0023° ($4 \times 10^{-5}$ rad).

### 3.4 Affect on detector coordinates

The differences depicted in Figures 2–4 are quite small. The LSI coordinates in the CALDB files define CCDs that are close to, but not quite, squares. To determine the extent to which the differences shown in Figures 2–4 affect the computation of the DETX and DETY coordinates, the following test was performed.

1. Three of the four corners are used to define a plane (e.g. $P_1$, $P_2$ and $P_4$).
2. The LSI coordinates $\mathbf{x}_{\rm LSI}$ of an event at $(\mathrm{CHIPX}, \mathrm{CHIPY}) = (x, y)$ are computed using
$$\mathbf{x}_{\rm LSI} = \mathbf{x}_i + f_x \left(\mathbf{x}_j - \mathbf{x}_i\right) + f_y \left(\mathbf{x}_k - \mathbf{x}_i\right), \tag{2}$$
where, for example, $\mathbf{x}_i$, $\mathbf{x}_j$ and $\mathbf{x}_k$ are the LSI coordinates associated with points $P_1$, $P_4$ and $P_2$, respectively, $f_x = (x - 0.5)/1024$ and $f_y = (y - 0.5)/1024$.
3. The coordinates DETX and DETY are computed using the coordinates $\mathbf{x}_{\rm LSI}$.
4. Steps 2 and 3 are repeated using the $(x, y)$ coordinates for each pixel of each CCD. This process yields a set of 10,485,760 DETX and DETY coordinates.
5. Steps 1–4 are repeated for the other three sets of three corners, yielding three more sets of 10,485,760 DETX and DETY coordinates.
6. The differences between the first and second, first and third and first and fourth sets of DETX and DETY coordinates are computed.

The largest difference between the first and second, first and third and first and fourth sets of DETX and DETY coordinates is plotted for each CCD in Figure 5. A comparison of Figures 2–4 with Figure 5 suggests that the technique I use to compute the DETX and DETY coordinates is more sensitive to the inaccuracies of the angles $\theta$ than to the inaccuracies of the side lengths $l$ or the nonplanarity of the corners. However, the differences between the DETX and DETY coordinates computed using one set of three corners differs by more than 0.042 pixel from the DETX and DETY coordinates computed using any other set of three corners. Since differences between the sky coordinates should be comparable to differences between the detector coordinates, the differences shown in Figures 2–4 should not significantly affect the computation of the X and Y coordinates.

## 4 Summary

While Figures 2–4 show that the LSI coordinates of the ACIS CCDs in the CALDB instrument geometry files do not define two-dimensional square detectors, these figures also show that the differences between a square detector and the CALDB data are small. Figure 5 shows that the inaccuracies in the CALDB data should not affect the computation of the detector (and, hence, sky) coordinates by more than about 0.04 pixel.

If the LSI coordinates are adjusted at some time in the future, it would be nice if the coordinates were modified to define two-dimensional square detectors. This can be achieved, for example, by calibrating six of the twelve coordinates of the four corners and computing the remaining six coordinates by requiring the corners to be coplanar, by requiring the lengths of each side to be the same as the calibrated length and by requiring the angles between adjacent sides to be right angles. In this fashion, the three conditions listed in section 2 will be satisfied to machine precision.

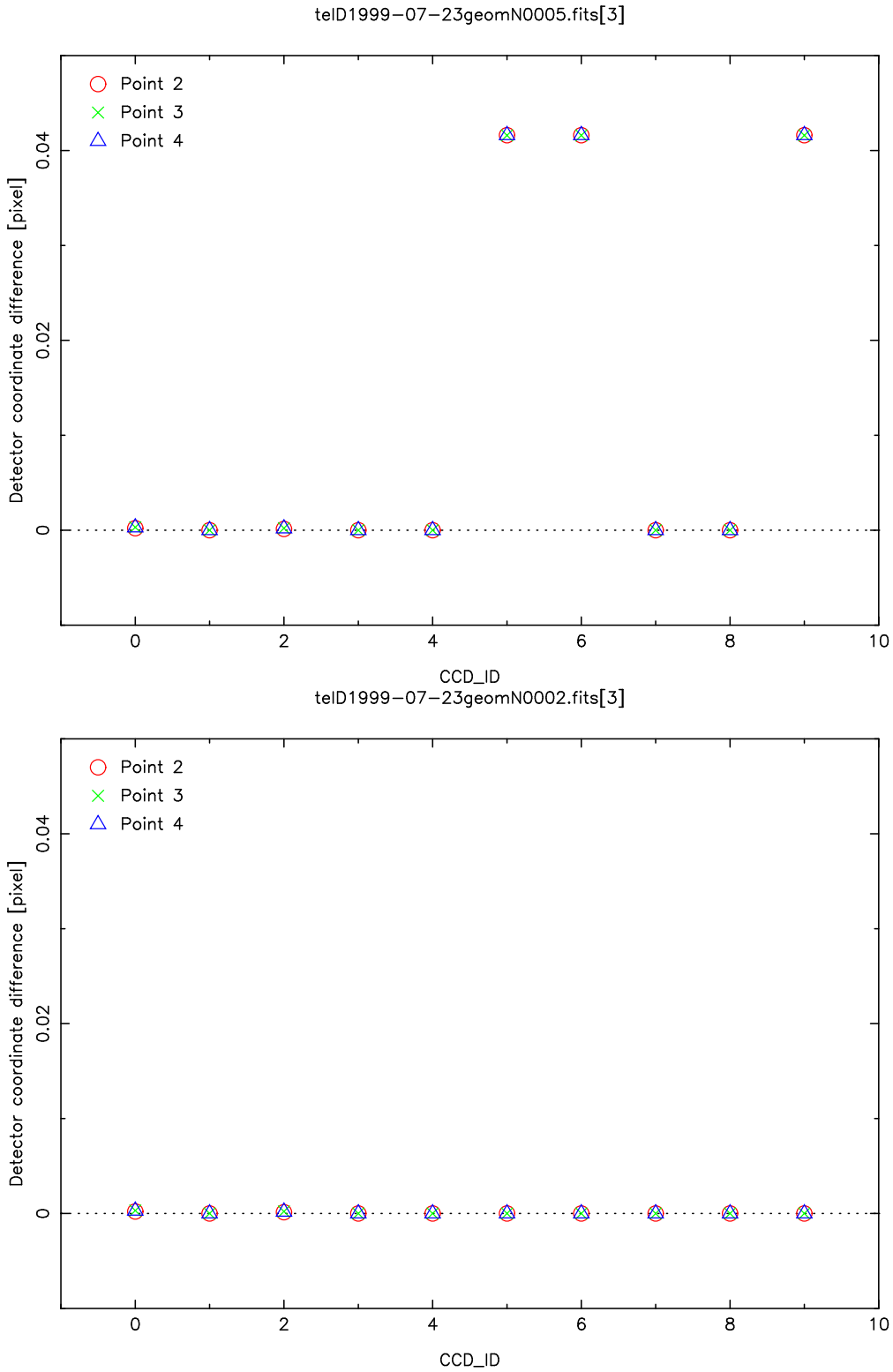


Figure 5: The largest difference between the DETX and DETY coordinates computed using points ($P_1$, $P_2$ and $P_3$) and ($P_1$, $P_2$ and $P_4$) ("Point 2"), using points ($P_2$, $P_3$ and $P_4$) and ($P_1$, $P_2$ and $P_4$) ("Point 3") and using points ($P_1$, $P_3$ and $P_4$) and ($P_1$, $P_2$ and $P_4$) ("Point 4"). The upper plot is for the coordinates in the CALDB file telD1999-07-23geomN0005.fits. The lower plot is for telD1999-07-23geomN0002.fits. The detector coordinates differ by no more than 0.042 pixel.